



# ALGEBRA

## बीजगणित

### SHEET-02

# CLASS NOTES

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# **CONCEPT OF PERFECT SQUARE**

$$* (a-3)^2 = 0$$

$$\Rightarrow (a-3) = 0$$

$$\Rightarrow \textcircled{a=3} \checkmark$$

$a=?$

$$Q. \underbrace{(a-2)}_0^2 + \underbrace{(b-4)}_0^2 = 0$$

$$\textcircled{a=2}$$

$$\textcircled{b=4}$$

$$\therefore \sqrt{3+4+9} \\ = 4$$

1. If  $(a - 3)^2 + (b - 4)^2 + (c - 9)^2 = 0$ ,

then  $\sqrt{a+b+c} = ?$

(a) -4

(c)  $\pm 4$

☒ (b) 4

(d)  $\pm 2$

$$a=3 \quad b=4 \quad c=9$$

$$\begin{array}{l|l} x^2 = 9 & x = \sqrt{9} \\ x = \pm \sqrt{9} & x = 3 \\ x = \pm 3 & \end{array}$$

2. If  $(a - 1)^2 + (b + 2)^2 + (c + 1)^2 = 0$ , then find  $2a - 3b + 7c = ?$

(a) 12

(b) -11

(c) 3

☒ (d) 1

$\rightarrow a=1 \quad b=-2 \quad c=-1$

$$\begin{aligned} & 2(1) - 3(-2) + 7(-1) \\ &= 2 + 6 - 7 \\ &= 1 \end{aligned}$$

3. If  $(a - 4)^2 + (b - 5)^2 + (c - 3)^2 = 0$ ,

then the value of  $\frac{a+b}{c}$  is:

(a) 0

✓ (b) 3

(c) 1

(d) -3



$\rightarrow a=4 \quad b=5 \quad c=3$

$$\therefore \frac{a+b}{c} = \frac{4+5}{3} = 3$$

4. If  $(x + y - z)^2 + (y + z - x)^2 + (z + x - y)^2 = 0$ ,  
then  $x + y + z = ?$

(a)  $\sqrt{3}$

(b)  $3\sqrt{3}$

(c) 3

☒ (d) 0

$$\begin{array}{r} \cancel{x} + \cancel{y} - z = 0 \\ + \quad y + \cancel{z} - \cancel{x} = 0 \\ + \quad z + \cancel{x} - \cancel{y} = 0 \\ \hline x + y + z = 0 \end{array}$$

$$* (a-4)^2 + (b-5)^2 + (c-3)^2 = 0$$

$$* a^2 + b^2 + c^2 = 2(4a + 5b + 3c) - 50$$

$$\begin{cases} a=4 \\ b=5 \\ c=3 \end{cases}$$

$$\frac{a+b}{c} =$$

$$\frac{a+b}{c}$$

$$a^2 + 16 - 8a + b^2 + 25 - 10b + c^2 + 9 - 6c = 0$$

$$\Rightarrow a^2 + b^2 + c^2 = 8a + 10b + 6c - 50$$

$$\Rightarrow a^2 + b^2 + c^2 = 2(4a + 5b + 3c) - 50$$

Condition

✓ ⊙ 2 (Variables)

⊙  $a = \frac{\text{Coefficient of } a}{\text{Coefficient of } a^2}$

$$\therefore a = \frac{4}{1} \quad b = \frac{5}{1} \quad c = \frac{3}{1}$$

$$\therefore \frac{a+b}{c} = \frac{4+5}{3} = 3 \checkmark$$

5. If  $a^2 + b^2 + c^2 = 2(a - b - c) - 3$ , then the value of  $4a - 3b + 5c$  is -

☒ (a) 2  
(c) 5

(b) 3  
(d) 6

$$a = \frac{1}{1} \quad b = -\frac{1}{1} \quad c = -\frac{1}{1}$$

$$\begin{aligned} \therefore 4(1) - 3(-1) + 5(-1) \\ = 4 + 3 - 5 \\ = 2 \end{aligned}$$

6. If  $a^2 + b^2 + 49c^2 + 18 = 2(b - 28c - a)$  then the value of  $(a + b - 7c)$  is:

✓ SSC CGL 12/04/2022 (Shift- 2)

✓ (a) 4

(b) 3

(c) 2

(d) 1

$$a = -\frac{1}{1} \quad b = \frac{1}{1} \quad c = \frac{-28 \times 4}{49 \times 7}$$

$$\begin{aligned} \therefore a + b - 7c &= -\cancel{1} + \cancel{1} - 7\left(\frac{-4}{7}\right) \\ &= (+4) \end{aligned}$$

7.  
H.W

If  $a^2 + b^2 + 49c^2 + 18 = 2(b - 28c - a)$ , then the value of  $(a - b - 7c)$  is:

✓ SSC CGL 18/04/2022 (Shift- 2)

(a) 2

(b) 1

(c) 3

(d) 4

8. ✓ If  $a^2 + b^2 + 49c^2 + 18 = 2(b + 28c - a)$ , then the value of  $(2a - b + 7c)$  is:

**SSC CGL 19/04/2022 (Shift- 2)**

(a) - 3

(c) - 4

✓ (b) 1

(d) 5

9. If  $a^2 + b^2 + c^2 + 84 = 4(a - 2b + 4c)$ , then  $\sqrt{ab - bc + ca}$  is equal to:

✓ **SSC CPO 24/11/2020 (Shift-2)**

(a)  $4\sqrt{10}$

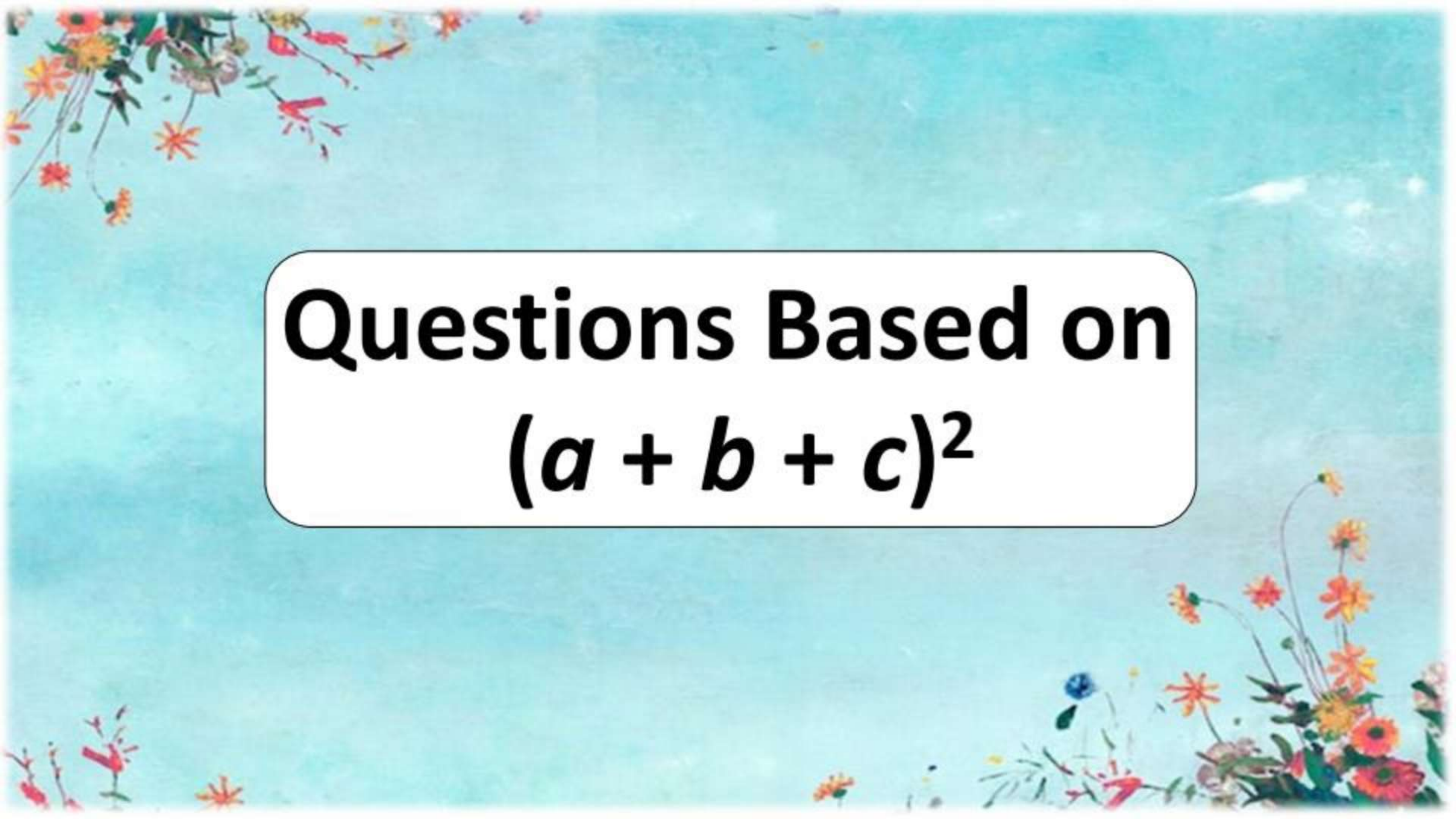
(b)  $\sqrt{10}$

(c)  $5\sqrt{10}$

✓ (d)  $2\sqrt{10}$

$a = \frac{2}{1} \quad b = \frac{-4}{1} \quad c = \frac{8}{1}$

$$\sqrt{2 \times (-4) - (-4) \times 8 + 8 \times 2}$$
$$= \sqrt{-8 + 32 + 16} = \sqrt{40} = 2\sqrt{10}$$



# Questions Based on $(a + b + c)^2$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ac)$$

m-1

10. If  $(a + b + c) = 16$ , and  $(a^2 + b^2 + c^2) = 90$ , find the value of  $(ab + bc + ca)$ .

SSC CGL 14/07/2023 (Shift-01)

(a) 84

☒ (b) 83

(c) 82

(d) 81

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$\Rightarrow (16)^2 = 90 + 2k$$

$$\Rightarrow \frac{166}{2} = k$$

$$\Rightarrow \underline{83 = k}$$

m-2

10. If  $(a + b + c) = 16$ , and  $(a^2 + b^2 + c^2) = 90$ , find the value of  $(ab + bc + ca)$ .

SSC CGL 14/07/2023 (Shift-01)

(a) 84

✓ (b) 83

(c) 82

(d) 81

$(a+b+c)=16$      $(a^2+b^2+c^2)=90$      $ab+bc+ca=?$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow 256 = 90 + 2ab + 2bc + 2ca$

$\Rightarrow 166 = 2ab + 2bc + 2ca$   
83

11. If  $a + b - \cancel{c} = 5$  and  $ab - \cancel{bc} - \cancel{ac} = 10$ , then find the value of  $a^2 + b^2 + \cancel{c^2}$ .

**SSC CGL 18/04/2022 (Shift- 01)**

(a) 15

(b) 45

(c) 5

(d) 40

$$a+b=5 \quad ab=10 \quad a^2+b^2$$

$$\therefore (a+b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow 25 = a^2 + b^2 + 20$$

$$\Rightarrow S = a^2 + b^2$$

$$\# (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$$

$$\# (a+b-c)^2 = a^2+b^2+c^2+2ab-2bc-2ac$$

$$\# (a-b+c)^2 = a^2+b^2+c^2-2ab-2bc+2ac.$$

11. If  $a + b - c = 5$  and  $ab - bc - ac = 10$ , then find the value of  $a^2 + b^2 + c^2$ .

**SSC CGL 18/04/2022 (Shift- 01)**

(a) 15

(b) 45

☒ (c) 5

(d) 40

$$(a+b-c)^2 = \underbrace{a^2+b^2+c^2} + 2ab - 2bc - 2ac$$

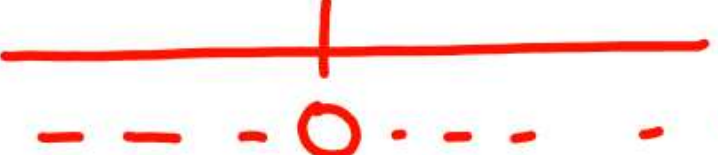
$$\Rightarrow (5)^2 = k + 2(ab - bc - ac)$$

$$\Rightarrow 25 = k + 2 \times 10$$

$$\Rightarrow 5 = k$$

12. If  $a^2 + b^2 + \cancel{c^2} = 6.25$  and  $(ab + \cancel{bc} + \cancel{ca}) = 0.52$ ,  
what is the value of  $(a + b + \cancel{c})$ , if  $(a + b + \cancel{c}) < 0$ ?

**SSC CGL 11/04/2022 (Shift- 03)**

-ve 

~~(a)  $\pm 2.7$~~

☒ (b)  $- 2.7$

(c)  $- 2.8$

~~(d)  $\pm 2.8$~~

$$a^2 + b^2 = 6.25 \quad ab = 0.52 \quad a + b = ?$$

$$\Rightarrow (a + b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow (a + b)^2 = 6.25 + 1.04$$

$$\Rightarrow (a + b)^2 = 7.29$$

$$\Rightarrow (a + b) = \pm 2.7$$

ans  $a + b = -2.7$

$$\therefore a+b+c < 0$$

$$\therefore a+b+c = -2.7$$

12. If  $a^2 + b^2 + c^2 = 6.25$  and  $(ab+bc+ca) = 0.52$ , what is the value of  $(a+b+c)$ , if  $(a+b+c) < 0$ ?

**SSC CGL 11/04/2022 (Shift- 03)**

(a)  $\pm 2.7$

☒ (b)  $- 2.7$

(c)  $- 2.8$

(d)  $\pm 2.8$

$$\begin{aligned} (a+b+c)^2 &= a^2 + b^2 + c^2 + 2(ab+bc+ca) \\ &= 6.25 + 2 \times 0.52 \end{aligned}$$

$$\begin{aligned} \therefore (a+b+c)^2 &= 6.25 + 1.04 \\ &= 7.29 \end{aligned}$$

$$(a+b+c) = \pm \sqrt{7.29} = \pm 2.7$$

. If  $x^9 + y^6 + z^4 = 19$ ,  $x^2 + y^2 + z^2 = 133$  and  $xz = y^2$ , then the difference between  $z$  and  $x$  is:

**SSC CHSL 05/08/2021 (Shift- 01)**

(a) 3

(b) 8

(c) 5

(d) 9

✓ CPO  
✓ CGL  
✓ MTS  
✓ CHSL

$$\therefore z - x = 6 - 3 = \underline{3}$$

13. If  $x^9 + y^3 + z^1 = \underline{13}$ ,  $x^8 + y^9 + z^1 = \underline{91}$  and  $xz = y^2$ , then the difference between  $z$  and  $x$  is:

**SSC CHSL 05/08/2021 (Shift- 01)**

(a) 3

✓ (b) 8

(c) 5

(d) 9

✓ CPO  
✓ CGL  
✓ MTS  
✓ CHSL

$$|z - x| = |1 - 9| = 8$$

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$$

$$\Rightarrow (13)^2 = 91 + 2(xy + yz + zx)$$

$$\Rightarrow 169 = 91 + 2y(x+z+y)$$

$$\Rightarrow 3 \cancel{78} = \cancel{2}y \times \cancel{13}$$

$$\therefore 3 = y$$

$$\therefore \begin{array}{l|l|l} x+y+z=13 & x^2+y^2+z^2=91 & xz=y^2 \\ x+z=10 & x^2+z^2=82 & xz=9 \end{array}$$

13. If  $x + y + z = 13$ ,  $x^2 + y^2 + z^2 = 91$  and  $xz = y^2$ , then the difference between  $z$  and  $x$  is:

SSC CHSL 05/08/2021 (Shift- 01)

(a) 3

☒ (b) 8

(c) 5

(d) 9

✓ CPO  
✓ CGL  
✓ MTS  
✓ CHSL

$$\begin{array}{l} 3 \times 3 \\ 1 \times 9 \end{array} \rightarrow \text{ans} = 8$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ac)$$

$$\Rightarrow 9 = 6 + 2abc$$

$$\Rightarrow 3 = 2abc$$

$$\Rightarrow \frac{3}{2} = abc$$

14. If  $a + b + c = 3$ ,  $a^2 + b^2 + c^2 = 6$  and  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$ ,

Here,  $a$ ,  $b$  &  $c$  are non-negative, then  $abc = ?$

(a)  $\frac{2}{3}$

(b)  $\frac{3}{2}$

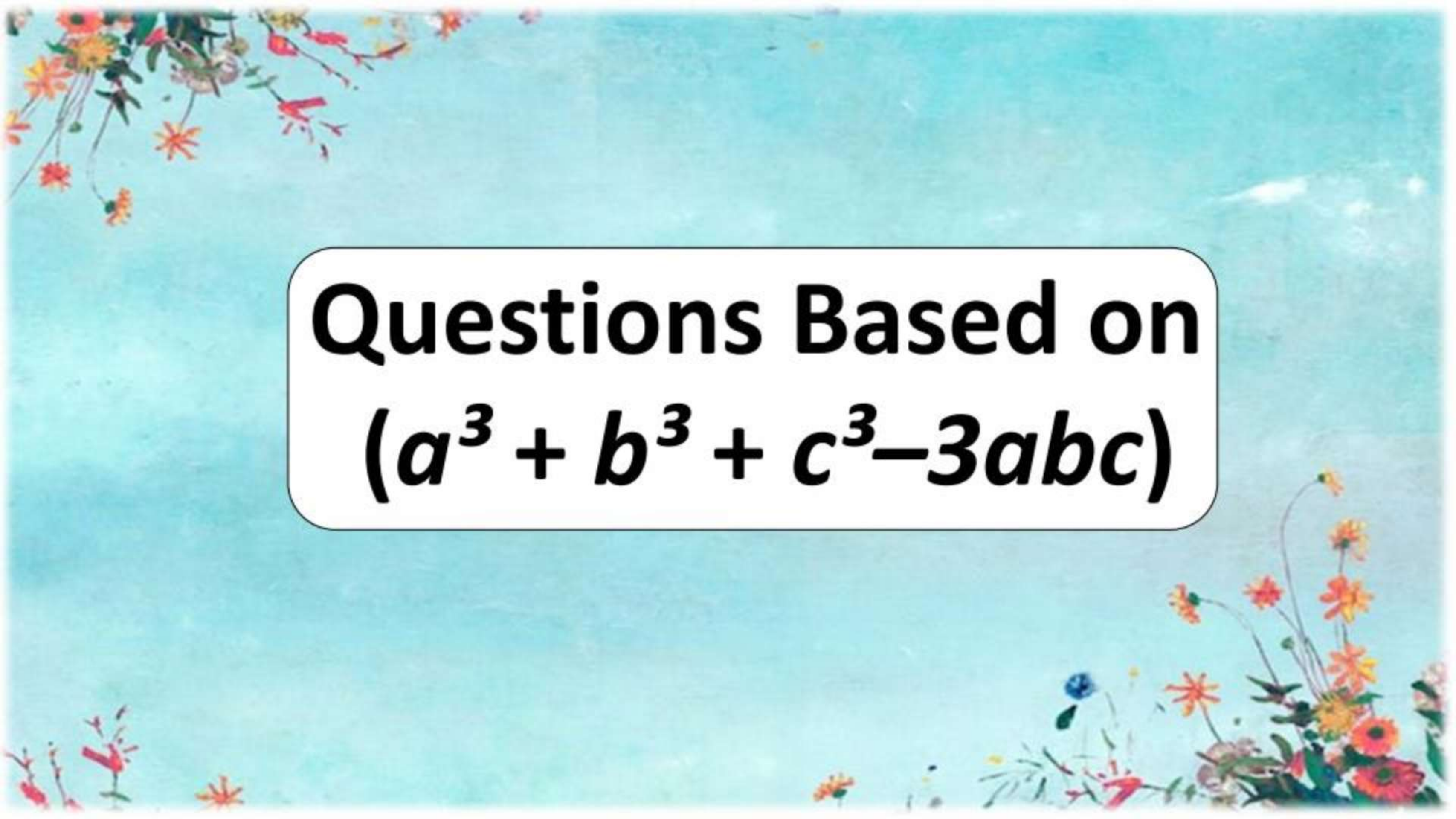
(c)  $\frac{1}{2}$

(d)  $\frac{1}{3}$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$$

$$\Rightarrow \frac{bc+ac+ab}{abc} = 1$$

$$\Rightarrow abc = ab+bc+ac$$



# Questions Based on $(a^3 + b^3 + c^3 - 3abc)$

$a^3+b^3+c^3-3abc$  —→  $\underbrace{(a+b+c)} \left[ \underbrace{a^2+b^2+c^2} - \underbrace{ab-bc-ac} \right] \quad (F-1)$

—→  $\underbrace{(a+b+c)} \left[ \underbrace{(a+b+c)^2} - 3 \underbrace{(ab+bc+ac)} \right] \quad (F-2)$

—→  $\frac{(a+b+c)}{2} \left[ 3 \underbrace{(a^2+b^2+c^2)} - \underbrace{(a+b+c)^2} \right] \quad (F-3)$

—→  $\frac{(a+b+c)}{2} \left[ (a-b)^2 + (b-c)^2 + (c-a)^2 \right]$

—→ (F-4) for big numbers.

**15. If  $a + b + c = 6$ ,  $a^2 + b^2 + c^2 = 14$  and  $ab + bc + ca = 11$ , then what is the value of  $a^3 + b^3 + c^3 - 3abc$  ?**

**SSC CGL 02/12/2022 (Shift-03)**

(a) 31

(b) 12

☒ (c) 18

(d) 42

$$\begin{aligned}\therefore a^3 + b^3 + c^3 - 3abc &= (a+b+c)[a^2+b^2+c^2 - ab - bc - ca] \\ &= (a+b+c)[a^2+b^2+c^2 - (ab+bc+ca)] \\ &= 6[14-11] \\ &= 6 \times 3 \\ &= \underline{18}\end{aligned}$$

16. If  $a + b + c = 5$ ,  $a^3 + b^3 + c^3 = 85$  and  $abc = 25$ , then find the value of  $a^2 + b^2 + c^2 - ab - bc - ca$ .

(a) 2

(b) 4

(c) 6

(d) 8

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)[a^2 + b^2 + c^2 - ab - bc - ca]$$

$$\Rightarrow 85 - 3 \times 25 = 5[R]$$

$$\Rightarrow 210 = 5R$$

$$\textcircled{2 = R}$$

17. Factorize the given algebraic expression.

$$(x+3y+4z)(x^2+9y^2+16z^2-3xy-12yz-4xz)$$

$$x^3 + 27y^3 + 64z^3 - 36xyz$$

$$(x)^3 + (3y)^3 + (4z)^3 - 3 \times x \times 3y \times 4z$$

~~(a)  $(x + 3y + 4z)(x^2 + 9y^2 + 16z^2 + 3xy + 12yz + 4xz)$~~

~~(b)  $(x + 3y + 4z)(x^2 + 9y^2 + 16z^2 - 12xy^2 - 3yz - 4xz)$~~

~~(c)  $(x - 3y - 4z)(x^2 + 9y^2 + 16z^2 - 3xy - 12yz - 4xz)$~~

(d)  $(x + 3y + 4z)(x^2 + 9y^2 + 16z^2 - 3xy - 12yz - 4xz)$

18. What is the value of

$$\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ac}$$
$$= \frac{(a+b+c) \cancel{[a^2+b^2+c^2-ab-bc-ac]}}{\cancel{[a^2+b^2+c^2-ab-bc-ac]}}$$
$$= 5$$

$$\frac{\left[ \overset{a}{(2.247)^3} + \overset{b}{(1.730)^3} + \overset{c}{(1.023)^3} \right] - 3 \times 2.247 \times 1.730 \times 1.023}{\left[ (2.247)^2 + (1.730)^2 + (1.023)^2 - 2.247 \times 1.730 - 1.730 \times 1.023 - 2.247 \times 1.023 \right]}$$

(a) 1.730

(b) 4

✓ (c) 5

(d) 5.247

$$\begin{array}{r} 2.247 \\ 1.730 \\ 1.023 \\ \hline 5.000 \end{array}$$

**19. If  $a + b + c = 5$  and  $ab + bc + ca = 7$ , then the value of  $a^3 + b^3 + c^3 - 3abc$  is:**

**SSC CPO 05/10/2023 (Shift-01)**

**(a) 20**

**(b) 25**

**(c) 15**

**(d) 30**

$$\begin{aligned}a^3 + b^3 + c^3 - 3abc &= (a+b+c) [(a+b+c)^2 - 3(ab+bc+ca)] \\&= 5 [25 - 3 \times 7] \\&= 5 \times 4 \\&= 20\end{aligned}$$

19. If  $a + b + \cancel{c} = 5$  and  $ab + \cancel{bc} + \cancel{ca} = 7$ , then the value of  $a^3 + b^3 + \cancel{c^3} - \cancel{3abc}$  is:

**SSC CPO 05/10/2023 (Shift-01)**

(a) 20

(b) 25

(c) 15

(d) 30

$$a+b=5 \quad ab=7$$

$$a^3+b^3=?$$

$$\begin{aligned} a^3+b^3 &= (a+b)(a^2+b^2-3ab) \\ &= 5(25-3 \times 7) \\ &= 5 \times 4 \\ &= 20 \end{aligned}$$

20. If  $(a + b + c) = 14$ , and  $(a^3 + b^3 + c^3 - 3abc) = 98$ , find the value of  $(ab + bc + ca)$ .

**SSC CGL 26/07/2023 (Shift-04)**

(a) 60

(b) 64

(c) 65

✓ (d) 63

$$\underline{a^3 + b^3 + c^3 - 3abc} = \underline{(a+b+c)} [ (a+b+c)^2 - 3(ab+bc+ca) ]$$

$$\cancel{7} \cancel{98} = \cancel{14} [196 - 3k]$$

$$\Rightarrow \cancel{3k} = \cancel{189}$$
$$k = 63$$

21. If  $(a + b + c) = 12$ , and  $(a^2 + b^2 + c^2) = 50$ , find the value of  $(a^3 + b^3 + c^3 - 3abc)$ .

**SSC CGL 19/07/2023 (Shift-02)**

✓ (a) 36  
(c) 42

(b) 24  
(d) 48

$$\begin{aligned} a^3 + b^3 + c^3 - 3abc &= \frac{(a+b+c)}{2} [3(a^2 + b^2 + c^2) - (a+b+c)^2] \\ &= \frac{6}{2} [3 \times 50 - 144] \\ &= 6 \times 6 \\ &= 36 \end{aligned}$$

22. If  $x + y + z = 2$ ,  $x^3 + y^3 + z^3 - 3xyz = 74$ ,  
then  $(x^2 + y^2 + z^2)$  is equal to :

**SSC CGL 23/08/2021 (Shift- 03)**

(a) 22

(b) 29

(c) 26

(d) 24

$$x^3 + y^3 + z^3 - 3xyz = \frac{(x+y+z)}{2} [3(x^2 + y^2 + z^2) - (x+y+z)^2]$$

$$\Rightarrow 74 = \frac{2}{2} [3x - 4]$$

$$\Rightarrow 74 = 3x - 4$$

$$\Rightarrow 78 = 3x$$

26

$$\therefore \sqrt[3]{xyz} = -6$$

23. If  $x + y + z = 13$ ,  $x^2 + y^2 + z^2 = 133$  and  $x^3 + y^3 + z^3 = 847$ , then the value of  $\sqrt[3]{xyz}$  is:

SSC CPO 24/11/2020 (Shift- 02)

(a) 8

(b) 7

(c) -9

✓ (d) -6

$$(x^3 + y^3 + z^3 - 3xyz) = \frac{(x+y+z)}{2} [3(x^2 + y^2 + z^2) - (x+y+z)^2]$$

$$\Rightarrow 847 - 3xyz = \frac{13}{2} [3 \times 133 - 13^2]$$

$$\Rightarrow 847 - 3xyz = \frac{13}{2} [399 - 169]$$

$$\Rightarrow 847 - 3xyz = 13 \times 115$$

$$\Rightarrow -3xyz = 648 \text{ or } 216$$

$$\Rightarrow xyz = -216$$

$$Q. \quad a+b+c=10 \quad a^2+b^2+c^2=80 \quad a^3+b^3+c^3-3abc=?$$

Ans

$$a^3+b^3+c^3-3abc = \frac{(a+b+c)}{2} [3(a^2+b^2+c^2) - (a+b+c)^2]$$

$$= \frac{10}{2} [3 \times 80 - 100]$$

$$= \frac{10}{2} \times 140$$

$$= \underline{700}$$

24. If  $a = 355$ ,  $b = 356$ ,  $c = 357$  find  $a^3 + b^3 + c^3 - 3abc = ?$

**SSC CHSL 15/10/2020 (Shift- 01)**

(a) 3208

(b) 3202

(c) 3206

☒ (d) 3204

$$\begin{aligned} a^3 + b^3 + c^3 - 3abc &= \frac{(a+b+c)}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] \\ &= \frac{1068}{2} [1 + 1 + 4] \\ &= \frac{1068}{2} \times 6 = 3204 \end{aligned}$$

\*  $a = \boxed{355}$   $b = 356$   $c = 357$

$d = 1$  (distance between  $a$  and  $b$ )

$2d$  (distance between  $a$  and  $c$ )

$d$  (distance between  $b$  and  $c$ )

$$a^3 + b^3 + c^3 - 3abc = (a+b+c) \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= (\cancel{b-d} + b + b + \cancel{d}) \frac{1}{2} [(-d)^2 + (-d)^2 + (2d)^2]$$

$$= 3b \times \frac{1}{2} [d^2 + d^2 + 4d^2]$$

$$= 3b \times 3d^2$$

$$= \underline{\underline{9bd^2}}$$

24. If  $a = 355$ ,  $b = 356$ ,  $c = 357$  find  $a^3 + b^3 + c^3 - 3abc = ?$

**SSC CHSL 15/10/2020 (Shift- 01)**

(a) 3208

(b) 3202

(c) 3206

✓ (d) 3204

$$\begin{aligned} a^3 + b^3 + c^3 - 3abc &= 9bcd^2 \\ &= 9 \times 356 \times 1 \\ &= \underline{3204} \end{aligned}$$

25. If  $x = 222$ ,  $y = 223$  and  $z = 224$ , then find the value of  $x^3 + y^3 + z^3 - 3xyz$ .

**SSC CGL 12/12/2022 (Shift-04)**

☒ (a) 2007

(b) 2004

(c) 2006

(d) 2005

$$\begin{aligned}\text{Ans} &= 9bd^2 \\ &= 9 \times 223 \times 1 \\ &= \underline{2007}\end{aligned}$$

26. Find  $a^3 + b^3 + c^3 - 3abc$  if  $a = 1001$ ,  
 $b = 1004$  and  $c = 1007$ .

(a) 80235

(b) 81234

(c) 79356

✓ (d) 81324

$$a=1001 \quad b=1004 \quad c=1007$$

$$\begin{aligned} \text{Ans} &= 9bd^2 \\ &= 9 \times 1004 \times 9 \\ &= 81 \times 1004 \\ &\therefore 81324 \end{aligned}$$

$$\begin{matrix} 8032 \\ 1002 \end{matrix}$$

27. If  $x = z = \underline{225}$ , and  $y = \underline{226}$  then  $x^3 + y^3 + z^3 - 3xyz = ?$

(a) 765

☒ (b) 676

(c) 576

(d) 674

$$x^3 + y^3 + z^3 - 3xyz = \frac{(x+y+z)}{2} [(x-y)^2 + (y-z)^2 + (z-x)^2]$$

$$= \frac{676}{2} [\cancel{1} + \cancel{1} + 0]$$

$$= 676$$

$$\cancel{(a+b+c)} [a^2+b^2+c^2-ab-bc-ac] = \frac{\cancel{(a+b+c)}}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$\therefore a^2+b^2+c^2-ab-bc-ac = \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

But  $a, b, c$  are in A.P  
 then  $\rightarrow = 3d^2$

28. If  $a = 2022$ ,  $b = 2021$  and  $c = 2020$ , then value of  $a^2 + b^2 + c^2 - ab - bc - ca$  is:

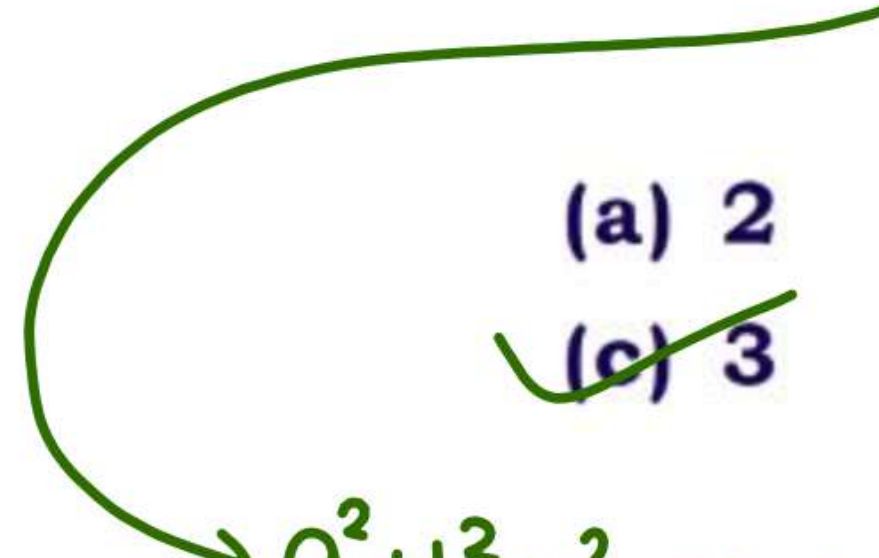
**SSC CHSL 26/05/2022 (Shift- 03)**

(a) 2

(b) 4

☒ (c) 3

(d) 1


$$\begin{aligned} \rightarrow a^2 + b^2 + c^2 - ab - bc - ca &= \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] \\ &= 3d^2 \\ &= 3 \times (1)^2 \\ &= 3 \end{aligned}$$

29. If  $x = 2015$ ,  $y = 2014$  and  $z = 2013$  then the value of  $x^2 + y^2 + z^2 - xy - yz - zx$ ,

~~(a) 3~~

(c) 5

(b) 4

(d) 6

$$\begin{aligned} &= 3d^2 \\ &= 3 \times 1 \\ &= 3 \end{aligned}$$

30. If  $a = 101$ ,  $b = 102$  and  $c = 103$ , then  $a^2 + b^2 + c^2 - ab - bc - ca$

SSC CGL 24/07/2023 (Shift-01)

(a) 2

(b) 4

☒ (c) 3

(d) 6

→ ans =  $3d^2$   
 $= 3 \times 1$   
 $= 3$

31. If  $x = 32$ ,  $y = 33$  and  $z = 35$ , then evaluate the expression  $x^3 + y^3 + z^3 - 3xyz$ .

**SSC CHSL 01/06/2022 (Shift- 03)**

(a) 1120

(b) 1000

(c) 900

✓ (d) 700

$$\begin{aligned}x^3 + y^3 + z^3 - 3xyz &= \frac{(x+y+z)}{2} [(x-y)^2 + (y-z)^2 + (z-x)^2] \\&= \frac{100}{2} [1 + 4 + 9] \\&= \frac{100 \times 14}{2} = 700\end{aligned}$$

$$\cancel{(a+b+c)} [a^2+b^2+c^2-ab-bc-ac] = \frac{\cancel{(a+b+c)}}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$\therefore a^2+b^2+c^2-ab-bc-ac = \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

But  $a, b, c$   $\left\{ \begin{array}{l} \text{are in A.P} \\ \text{then} \end{array} \right. \Rightarrow = 3d^2$

32. If  $ab + bc + ca = 8$  and  $a^2 + b^2 + c^2 = 20$ ,

then possible value of  $\frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$  is :

SSC CGL 06/06/2019 (Shift- 01)

- (a) ☒ 72  
(c) 84

- (b) 56  
(d) 80

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$
$$\therefore (a+b+c)^2 = 20 + 2 \times 8$$
$$\therefore (a+b+c) = 6$$

$$(a+b+c)[a^2+b^2+c^2-(ab+bc+ca)] = \frac{(a+b+c)}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$
$$= 6[20-8]$$
$$= 6 \times 12$$
$$= 72$$

$a=4$   $b=2$   $c=0$     32. If  $ab = 8$  and  $a^2 + b^2 = 20$ ,

$ab=8$      $a^2+b^2=20$   
 $\downarrow \downarrow$   
 $4 \quad 2$

then possible value of  $\frac{1}{2}(a+b)[(a-b)^2 + (b-a)^2 + (-a)^2]$  is :

**SSC CGL 06/06/2019 (Shift- 01)**

☒ (a) 72

(b) 56

(c) 84

(d) 80

$\rightarrow = \frac{1}{2}(a+b)[(a-b)^2 + b^2 + a^2]$   
 $= 3[4+20]$   
 $= 72$

33. If  $(a + b + c) = 19$ , and  $(a^2 + b^2 + c^2) = 155$ ,  
find the value of  $(a - b)^2 + (b - c)^2 + (c - a)^2$ .

**SSC CGL 18/07/2023 (Shift-02)**

~~(a) 104~~  
(c) 100

(b) 108  
(d) 98

$$\frac{(a+b+c)}{2} [3(a^2+b^2+c^2) - (a+b+c)^2] = \frac{(a+b+c)}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$\begin{aligned} \text{ans} &= (3 \times 155 - 361) \\ &= 465 - 361 \\ &= 104 \end{aligned}$$

34. If  $(a^3 + b^3 + c^3 - 3abc) = 405$ , and  $(a + b + c) = 15$ , find the value of  $(a - b)^2 + (b - c)^2 + (c - a)^2$ .

SSC CGL 20/07/2023 (Shift-01)

(a) 27

✓ (b) 54

(c) 18

(d) 45

$$a^3 + b^3 + c^3 - 3abc = \frac{(a+b+c)^3 - 3(a+b+c)(a^2+b^2+c^2) + 3(a+b+c)(ab+bc+ca) - 3abc}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$\Rightarrow \frac{27}{2} \times 15 = \frac{18}{2} \times R$$

$$\Rightarrow 54 = R$$

35. ✓ If  $(a^3 + b^3 + c^3 - 3abc) = 405$ , and  $(a - b)^2 + (b - c)^2 + (c - a)^2 = 54$ , find the value of  $(a + b + c)$ .

**SSC CGL 26/07/2023 (Shift-02)**

(a) 15

(b) 45

(c) 9

(d) 27

36. Simplify the following expression.

$$\frac{a^3 + b^3 + c^3 - 3abc}{a+b+c}$$

$$= \frac{(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]}{2(a+b+c)}$$

$$= \frac{1}{2} [25 + 9 + 4]$$

$$= \frac{1}{2} \times 38$$

$$= 19$$

$$\frac{(59 \times 59 \times \overset{a}{59}) + (54 \times 54 \times \overset{b}{54}) + (57 \times 57 \times \overset{c}{57}) - 3(59)(54)(57)}{(59 + 54 + 57)}$$

SSC CHSL 27/05/2022 (Shift- 01)

(a) 38

(b) 76

(c) 170

✓ (d) 19

**37. If  $a = 25$ ,  $b = 15$ ,  $c = -10$ , then**

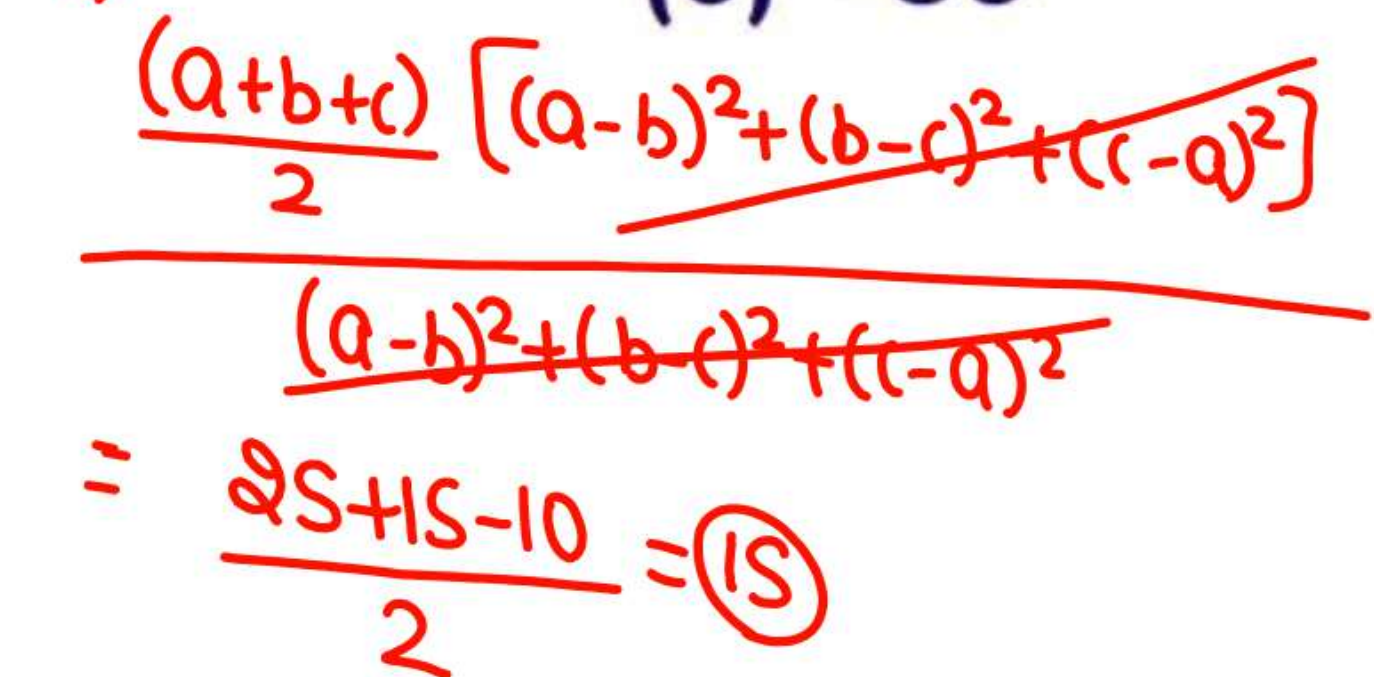
$$\frac{a^3 + b^3 + c^3 - 3abc}{(a-b)^2 + (b-c)^2 + (c-a)^2} = ?$$

(a) 30

(b) -15

(c) -30

(d) 15


$$\frac{(a+b+c)}{2} \frac{[(a-b)^2 + (b-c)^2 + (c-a)^2]}{(a-b)^2 + (b-c)^2 + (c-a)^2}$$
$$= \frac{25+15-10}{2} = \textcircled{15}$$

$$\underbrace{a^3+b^3+c^3-3abc}_0 = \frac{(a+b+c)}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

○

$$\frac{a+b+c}{2} = 0$$

$$\Rightarrow \boxed{a+b+c=0}$$

$$(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$a-b=0 \Rightarrow a=b$$

$$b-c=0 \Rightarrow b=c$$

$$c-a=0 \Rightarrow c=a$$

$$\boxed{a=b=c}$$

38. If  $a^3 - b^3 - c^3 - 3abc = 0$ , then

(a)  $a = b = c$

(b)  $a + b + c = 0$

(c)  $a + c = b$

✓ (d)  $a = b + c$

✓  $a^3 + b^3 + c^3 - 3abc = 0$  then  $a + b + c = 0$

✓  $a^3 - b^3 + c^3 + 3abc = 0$  then  $a - b + c = 0$

→  $a^3 - b^3 - c^3 - 3abc = 0$  then  $a - b - c = 0$   
 $\Rightarrow a = b + c$

39. What is the value of  $a^3 + b^3 + c^3$  if  $(a + b + c) = 0$ ?

**SSC CGL 14/07/2023 (Shift-02)**

(a)  $a^2 + b^2 + c^2 - 3abc$

(b) 0

(c)  $3abc$

(d)  $a^2 + b^2 + c^2 - ab - bc - ca$

If  $a+b+c=0$  then  $a^3+b^3+c^3-3abc=0$

$\Rightarrow a^3+b^3+c^3=3abc$

\*If  $a+b+c=0$

$$a^3+b^3+c^3-3abc=0$$

$$\Rightarrow a^3+b^3+c^3=3abc$$

If  $a+b+c=0$

$$\therefore a^3+b^3+c^3=3abc$$

40. Find the value of  $\underline{(1.6)^3} - \underline{(0.9)^3} - \underline{(0.7)^3}$ .

(a) 3.24

(b) -3.24

☒ (c) 3.024

(d) -3.024

$$\begin{aligned} \text{Ans} &= 3 \times (1.6) \times (-0.9) \times (-0.7) \\ &= 3.024 \end{aligned}$$

$$\begin{aligned} &1.6 - 0.9 - 0.7 \\ &= 1.6 - 1.6 \\ &= \underline{0} \end{aligned}$$

41. If  $(3x + 1)^3 + (x - 3)^3 + (4 - 2x)^3 + 6(3x + 1)(x - 3)(x - 2) = 0$ , then what is the value of  $x$ ?

SSC CHSL 11/07/2019 (Shift- 01)

(a) ☒ -1

(b)  $-\frac{1}{2}$

(c) 1

(d)  $\frac{1}{2}$

$$(3x+1)^3 + (x-3)^3 + (4-2x)^3 + 6(3x+1)(x-3)(x-2) = 0$$
$$a^3 + b^3 + c^3 - 3abc = 0$$

$$\therefore a+b+c=0$$

$$3x+1+x-3+4-2x=0$$

$$\Rightarrow 2x+2=0$$

$$\Rightarrow x=-1$$

$$\therefore x=-1$$

**SSC CHSL 11/07/2019 (Shift- 01)**

- $$(3x+1)^3 + (x-3)^3 + (4-2x)^3 + 6(3x+1)(x-3)(x-2) = 0$$
- $$a^3 + b^3 + c^3 - 3abc = 0$$
- $$\underline{\underline{a+b+c = 0}}$$

42. If  $(4x - 5)^3 + (x - 2)^3 + 27(2x - 5)^3 = 9(4x - 5)(x - 2)(2x - 5)$ , then the value of

$\left(x + \frac{3}{2}\right)$  will be:

$2 + \frac{3}{2}$   
 $= \frac{7}{2}$

SSC CHSL 05/08/2021 (Shift- 02)

(a)  $\frac{1}{2}$

(b)  $\frac{5}{2}$

(c)  $\frac{3}{2}$

(d)  $\frac{7}{2}$

$$(4x-5)^3 + (x-2)^3 + (3)^3(2x-5)^3 = 9(4x-5)(x-2)(2x-5)$$
$$\Rightarrow (4x-5)^3 + (x-2)^3 + (6x-15)^3 - 9(4x-5)(x-2)(2x-5) = 0$$

$$\therefore 4x-5 + x-2 + 6x-15 = 0$$

$$\Rightarrow 11x - 22 = 0 \Rightarrow x = 2$$

43. If  $(5x + 1)^3 + (x - 3)^3 + (2)(3x - 4)^3 = 6(5x + 1)(x - 3)(3x - 4)$ , then  $x$  is equal to:

✓ CGL Tier II 12/09/2019

✓ (a)  $\frac{5}{6}$

(b)  $\frac{1}{3}$

(c)  $\frac{2}{3}$

(d)  $\frac{3}{4}$

$(5x+1)^3 + (x-3)^3 + (6x-8)^3 = 6(5x+1)(x-3)(3x-4)$

$\Rightarrow 5x+1+x-3+6x-8=0$

$\Rightarrow 12x-10=0$

$\Rightarrow 6 \cancel{12}x = 10$

$\Rightarrow x = \frac{5}{6}$

44. Given that  $(5x - 3)^3 + (2x + 5)^3 + 27(4 - 3x)^3 = 9(3 - 5x)(2x + 5)(3x - 4)$ , then the value of  $(2x + 1)$  is :

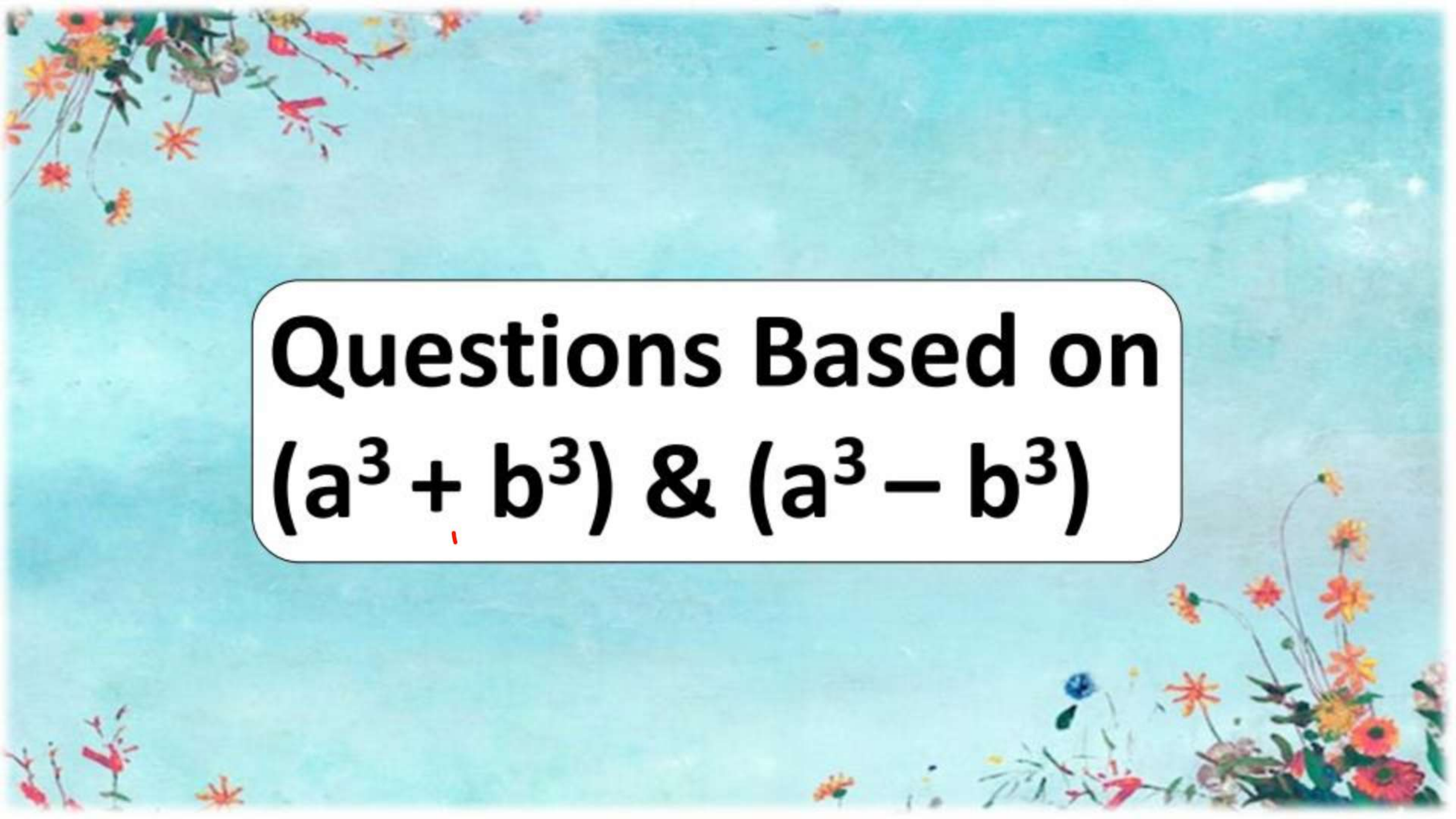
**SSC CGL Tier II 13/09/2019**

(a) - 13

(b) 15

(c) - 15

(d) 13



# Questions Based on $(a^3 + b^3)$ & $(a^3 - b^3)$

$$\begin{aligned} * a^3 + b^3 &= (a+b)(a^2 + b^2 - ab) \\ &= (a+b)[(a+b)^2 - 3ab] \end{aligned}$$

$$\begin{aligned} * a^3 - b^3 &= (a-b)[a^2 + b^2 + ab] \\ &\quad \text{or} \\ &= (a-b)[(a-b)^2 + 3ab] \end{aligned}$$

45. If  $a^3 - b^3 = 216$  and  $a - b = 6$ , then

$(a + b)^2 - ab$  is equal to :

SSC CPO 15 / 03 / 2019 (Shift- 03)

$$a^2 + b^2 + 2ab - ab \\ = a^2 + b^2 + ab$$

(a) 38

(b) 42

(c) 52

(d) 36

$$\therefore (a^3 - b^3) = (a - b) [a^2 + b^2 + ab]$$

$$36 \cancel{216} = \cancel{6} \times R$$

45. If  $a^3 = 216$  and  $a = 6$ , then

$(a + )^2$  is equal to :

$$6^2 = 36$$

**SSC CPO 15 /03/2019 (Shift- 03)**

(a) 38

(b) 42

(c) 52

✓ (d) 36

**46. If  $(a - b) = 4$  and  $ab = 2$ , then  $(a^3 - b^3)$  is equal to :**

**SSC CGL 13/06/2019 (Shift- 01)**

(a) 92

 (b) 88

(c) 84

(d) 80

$$\begin{aligned} a^3 - b^3 &= (a - b) [(a - b)^2 + 3ab] \\ &= 4 [16 + 6] \\ &= 88 \end{aligned}$$

47. If  $(a - b) = 9$ , and  $(a^3 - b^3) = 4410$ , find the value of  $ab$ .

SSC CHSL 02/08/2023 Shift-02

(a) 190

(b) 112

(c) 162

✓ (d) 136

$$a^3 - b^3 = (a - b) [(a - b)^2 + 3ab]$$

$$\Rightarrow \frac{4410}{9} = 81 + 3ab$$

$$\Rightarrow 490 = 81 + 3ab$$

$$\Rightarrow 409 = 3ab$$

$$136.33 = ab$$

**48. If  $a + b = 10$  and  $ab = 6$ , then the value of  $a^3 + b^3$  is:**

**SSC CPO 03/10/2023 (Shift-01)**

(a) 860

☒ (b) 820

(c) 800

(d) 840

$$\begin{aligned} a^3 + b^3 &= (a+b) [(a+b)^2 - 3ab] \\ &= 10 [100 - 18] \\ &= 820 \end{aligned}$$

49. If  $a^3 + b^3 = 432$  and  $a + b = 12$ ,  
then  $(a + b)^2 - 3ab$  is equal to :

**SSC CPO 16 /03/2019 (Shift- 02)**

(a) 42

(b) 52

✓ (c) 36

(d) 38

$$a^3 + b^3 = (a+b)[(a+b)^2 - 3ab]$$
$$36 \cancel{432} = 12 \times R$$

$$\therefore R = 36$$

50. If  $\frac{x}{y} + \frac{y}{x} = 1$  and  $x + y = 2$ , then the value of  $x^3 + y^3$  is:

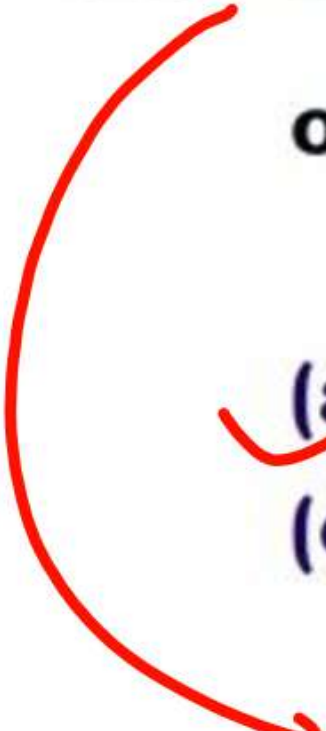
**SSC CPO 03/10/2023 (Shift-01)**

☒ (a) 0

(b) 1

(c) 3

(d) 2


$$\rightarrow \frac{x^2 + y^2}{xy} = 1 \Rightarrow x^2 + y^2 = xy \Rightarrow x^2 + y^2 - xy = 0$$

$$\begin{aligned}\therefore x^3 + y^3 &= (x+y)(x^2 + y^2 - xy) \\ &= 2 \times 0 \\ &= 0\end{aligned}$$

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$

$$\therefore (a^3+b^3) \div (a+b) = (a^2+b^2-ab)$$

$$\therefore (a^3-b^3) \div (a-b) = (a^2+b^2+ab)$$

51. If  $(8x^3 - 27y^3) \div (2x - 3y) = (Ax^2 + Bxy + Cy^2)$ , then the value of  $(2A + B - C)$  is :

SSC CGL 6 / 06 / 2019 (Shift- 1)

(a) 4

(b) 6

(c) 5

(d) 3

$$(8x^3 - 27y^3) \div (2x - 3y) = (Ax^2 + Bxy + Cy^2)$$
$$(a^3 - b^3) \div (a - b) = (a^2 + ab + b^2)$$

$$A=4, B=6, C=9$$

$$= 2A + B - C$$
$$= 8 + 6 - 9$$
$$= \textcircled{5}$$

51. If  $(8x^3 - 27y^3) \div (2x - 3y) = (Ax^2 + Bxy + Cy^2)$ , the value of  $(2A + B - C)$  is :

$$8 + 6 - 9 = 5$$

**SSC CGL 6 / 06 / 2019 (Shift- 1)**

(a) 4

(b) 6

☒ (c) 5

(d) 3

52. If  $2\sqrt{2}x^3 - 3\sqrt{3}y^3 = (\sqrt{2}x - \sqrt{3}y)(Ax^2 + By^2 + Cxy)$

then the value of  $A^2 + B^2 - C^2$  is :

**CGL Tier II (11/09/2019)**

(a) 11

(c) 19

☒ (b) 7

(d) 10

$\rightarrow 4 + 9 - 6$   
 $= 7$

53. If  $(8x^3 + 27y^3) \div (2x + 3y) = (Ax^2 + Bxy + Cy^2)$   
then the value of  $(5A + 4B + 3C)$  is :

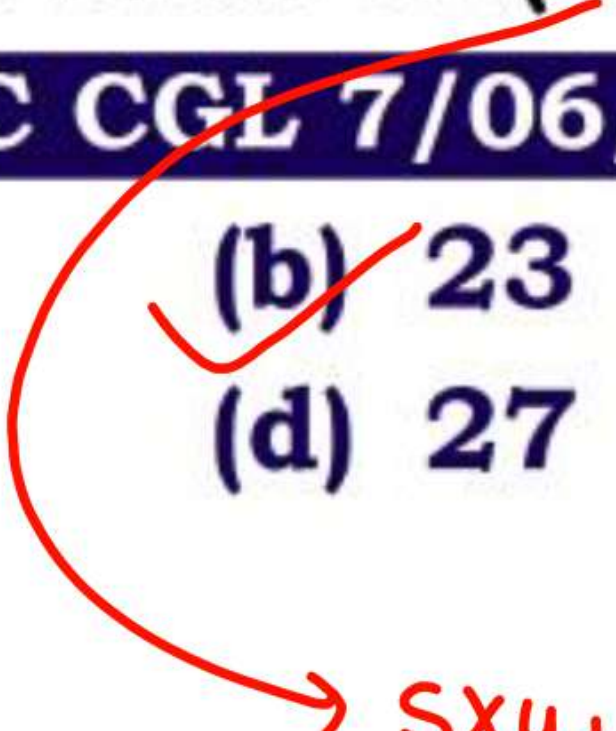
**SSC CGL 7/06/2019 (Shift- 03)**

(a) 26

(c) 24

(b) 23

(d) 27


$$\begin{aligned} &\rightarrow 5 \times 4 + 4(-6) + 3 \times 9 \\ &= 20 - 24 + 27 \\ &= 23 \end{aligned}$$

54. If  $5\sqrt{5}x^3 + 2\sqrt{2}y^3 = (Ax + \sqrt{2}y)(Bx^2 + 2y^2 + Cxy)$   
 $(\sqrt{5}x)^3 + (\sqrt{2}y)^3 = (\sqrt{5}x + \sqrt{2}y)(5x^2 + 2y^2 - \sqrt{10}xy)$   
 $xy)$ , then the value of  $(A^2 + B^2 - C^2)$  is

**CGL Tier II (13/09/2019)**

(a) 15

(c) 30

☒ (b) 20

(d) 40

$\rightarrow (\sqrt{5})^2 + 5^2 - (-\sqrt{10})^2$   
 $= 5 + 25 - 10$   
 $= 20$

$$\star (a^3+b^3) = (a+b)(a^2+b^2-ab)$$

$$\therefore (8x^3+27y^3) = (2x+3y)(4x^2+9y^2-6xy)$$

$$\star (2\sqrt{2}x^3+5\sqrt{5}y^3) = (\sqrt{2}x+\sqrt{5}y)(2x^2+5y^2-\sqrt{10}xy)$$

$$(2x+3y)^3 + (2x-3y)^3 = [(2x+3y) + (2x-3y)] \left\{ (2x+3y)^2 + (2x-3y)^2 - (2x+3y)(2x-3y) \right\}$$

$$\begin{aligned} \Rightarrow (2x+3y)^3 + (2x-3y)^3 &= 4x \{ 4x^2 + 9y^2 + 12xy + 4x^2 + 9y^2 - 12xy - (4x^2 - 9y^2) \} \\ &= 4x (4x^2 + 9y^2 + 4x^2 + 9y^2 - 4x^2 + 9y^2) \\ &= 4x (4x^2 + 27y^2) \end{aligned}$$

$$A+B-C \\ = 19+7-10 = \underline{16}$$

55. If  $27(x + y)^3 - 8(x - y)^3 = (x + 5y)(Ax^2 + By^2 + Cxy)$ , then what is the value of  $(A + B - C)$ ?

CGL 2019 Tier II (15/11/2020)

(a) 13

☒ (b) 16

(c) 18

(d) 11

$$\begin{aligned} \underbrace{(3x+3y)}_{a^3}^3 - \underbrace{(2x-2y)}_{b^3}^3 &= (x+5y)(Ax^2+By^2+Cxy) \\ a^3 - b^3 &= (a-b)((3x+3y)^2 + (2x-2y)^2 + \underbrace{(3x+3y)(2x-2y)}) \\ &= (9x^2 + 9y^2 + 18xy + 4x^2 + 4y^2 - 8xy + 6x^2 - 6xy + 6y^2) \\ &= (19x^2 + 7y^2 + 10xy) \\ &= (Ax^2 + By^2 + Cxy) \\ A=19, B=7, C=10 \end{aligned}$$

55. If  $27(x + y)^3 - 8(x - y)^3 = (x + 5y)(Ax^2 + By^2 + Cxy)$ , then what is the value of  $(A + B - C)$ ?

CGL 2019 Tier II (15/11/2020)

(a) 13

☒ (b) 16

(c) 18

(d) 11

$$(3x+3y)^3 - (2x-2y)^3 = (x+5y)(Ax^2 + By^2 + Cxy)$$

$$\{ (3x+3y)^2 + (2x-2y)^2 + (3x+3y)(2x-2y) \}$$

$$A = \frac{9}{19} \quad B = \frac{9}{7} \quad C = \frac{18}{10}$$

$$19 + 7 - 10 = 16$$

$$A - B - C \\ = 7 - 6 - 3 = -2$$

56. If  $8(x + y)^3 - (x - y)^3 = (x + 3y)(Ax^2 + Cy^2 + Bxy)$ , then the value of  $(A - B - C)$  is :

SSC CHSL 30/6/019 (Shift- 03)

☒ (a) - 2

(b) - 6

(c) 10

(d) 14

$$\begin{aligned} (2x+2y)^3 - (x-y)^3 &= (x+3y)(Ax^2 + Cy^2 + Bxy) \\ a^3 - b^3 &= (a-b) \{ (2x+2y)^2 + (x-y)^2 + (2x+2y)(x-y) \} \end{aligned}$$

|           |           |       |
|-----------|-----------|-------|
| $x^2$     | $y^2$     | $xy$  |
| 4 + 1 + 2 | 4 + 1 - 2 | 8 - 2 |
| A = 7     | C = 3     | B = 6 |