



ALGEBRA

बीजगणित

SHEET-03

CLASS NOTES

BY -ADITY RANJAN SIR

CONCEPT OF SYMMETRY

* $ab+bc \rightarrow$ same degree.

Symmetry

1. If $xy + yz + zx = 1$, then $\frac{1+y^2}{(x+y)(y+z)} = ?$

(a) 0

(c) 2

☒ (b) 1

(d) 3

$x = y = z$

$\rightarrow x^2 + x^2 + x^2 = 1$

$\Rightarrow 3x^2 = 1$

$\Rightarrow x^2 = \frac{1}{3}$

$$\frac{1+x^2}{2x \times 2x}$$
$$= \frac{1+x^2}{4x^2} = \frac{1+\frac{1}{3}}{4 \times \frac{1}{3}} = \frac{\frac{4}{3}}{\frac{4}{3}} = 1$$

2. If $x^2 + y^2 + z^2 = \overset{x \cdot x}{\underline{xy}} + \overset{x \cdot x}{\underline{yz}} + \overset{x \cdot x}{\underline{zx}}$ then the value of

$$\frac{3x^4 + 7y^4 + 5z^4}{5x^2y^2 + 7y^2z^2 + 3z^2x^2} \text{ is}$$

(a) 2

(c) 0

~~(b) 1~~

(d) -1

→ Check: $x=y=z$
 $3x^2 = 3x^2 \checkmark$

$$\rightarrow \frac{3x^4 + 7x^4 + 5x^4}{5x^4 + 7x^4 + 3x^4} = \frac{\cancel{15x^4}}{\cancel{15x^4}} = 1$$

3. If $a^2 + b^2 + c^2 = ab + bc + ca$ then $\frac{a+c}{b}$ is

(a) 0

(c) 1

~~(b) 2~~

(d) -1

→ Check: $a=b=c$

$$3a^2 = 3a^2$$

$$\frac{a+a}{a} = \frac{2a}{a}$$

put $x=y=z$

4. If $\frac{4x-3}{x} + \frac{4y-3}{y} + \frac{4z-3}{z} = 0$ then

$$\Rightarrow \frac{4x-3}{x} + \frac{4x-3}{x} + \frac{4x-3}{x} = 0$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \text{ is}$$

(a) 9

(b) 3

(c) 4

(d) 6

$$\Rightarrow \left(\frac{4x-3}{x}\right) \times 3 = 0$$

$$\Rightarrow 4x-3=0$$

$$\Rightarrow 4x=3$$

$$\Rightarrow x = \frac{3}{4}$$

$$\begin{aligned} & \frac{1}{x} + \frac{1}{x} + \frac{1}{x} \\ &= \frac{3}{x} = \frac{3 \times 4}{3} = 4 \end{aligned}$$

$$\left(\frac{2+a}{a} \right) \times 3 = 4$$

$$\Rightarrow 6+3a=4a$$

$$\Rightarrow \textcircled{6=a}$$

$$= \frac{3a^2}{a^3}$$

$$= \frac{3}{a} = \frac{1}{2}$$

5. If $\frac{2+a}{a} + \frac{2+b}{b} + \frac{2+c}{c} = 4$ then the value

of $\frac{(ab+bc+ca)}{abc}$ is

(a) 2

(b) 1

(c) 0

(d) $\frac{1}{2}$

put $a=b=c$

6. If $\frac{x + \overbrace{a^2} + 2\overbrace{c^2}}{\underbrace{b+c}} + \frac{x + \overbrace{b^2} + 2\overbrace{a^2}}{c+a} + \frac{x + \overbrace{c^2} + 2\overbrace{b^2}}{a+b} = 0$, find x

- (a) $a^2 + b^2 + c^2$ $3a^2$ (b) $-(a^2 + b^2 + c^2)$ $3a^2$
 (c) $a^2 + 2b^2 + c^2$ $4a^2$ (d) $-(a^2 + 2b^2 + c^2)$ $-4a^2$

$$\Rightarrow \left(\frac{x + 3a^2}{3a} \right) \times 3 = 0$$

$$\Rightarrow x + 3a^2 = 0$$

$$\Rightarrow \boxed{x = -3a^2}$$

$$a=b=c$$

7. If $\frac{x-a^2}{b^2+c^2} + \frac{x-b^2}{c^2+a^2} + \frac{x-c^2}{b^2+a^2} = 3$, find the value of x .

(a) ~~$a^2 + b^2 - c^2$~~ a^2

(b) $a^2 + b^2 + c^2$ $3a^2$

(c) ~~$a^2 - b^2 - c^2$~~ $-a^2$

(d) $a^2 + b^2$ $2a^2$

$$\left(\frac{x-a^2}{2a^2} \right) \times 3 = \frac{1}{1}$$

$$\Rightarrow x - a^2 = 2a^2$$

$$\Rightarrow x = 3a^2$$

put $a=b=c$

8. If $bc + ca + ab = abc$ then

$$\left\{ \frac{b+c}{bc(a-1)} + \frac{c+a}{ca(b-1)} + \frac{a+b}{ab(c-1)} \right\} = ?$$

(a) 0

☒ (b) 1

(c) 2

(d) 3

~~$3a^2 = a^3$~~

$a=3$

$$\begin{aligned} & \frac{2a}{a^2(a-1)} \times 3 \\ &= \frac{2 \times 3 \times 3}{9 \times 2} = 1 \end{aligned}$$

put $a=b=c$

9. If $a^2 = b + c$, $b^2 = c + a$, $c^2 = a + b$, then

$a^2 = 2a$
 $a=2$

$$\left\{ \frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} \right\}$$

(a) 0

(c) 2

☒ (b) 1

(d) 3

$$= \left(\frac{1}{1+a} \right) \times 3$$

$$= \frac{1}{3} \times 3$$

put $a=b=c$

$$a + \frac{1}{a} = 1$$

$$\Rightarrow a^2 + 1 = a \Rightarrow (a^2 - a + 1) = 0$$

$$\Rightarrow (a+1)(a^2 - a + 1) = 0$$

$$\Rightarrow a^3 + 1^3 = 0$$

$$\Rightarrow a^3 = -1$$

2017, 2023 Mains.

10. If a, b, c are non zero, $a + \frac{1}{b} = 1$ & $b + \frac{1}{c} = 1$

then (i) abc is $a^3 = -1$ (ii) $a + \frac{1}{a}$ is $= 1$

(a) ☒ -1, 1

(b) ☐ 3, -1

(c) ☐ -3, 1

(d) ☐ 1, 1

M-1 (Basic)

$$a^x = (x+y+z)^y$$

$$a^y = (x+y+z)^z$$

$$a^z = (x+y+z)^x$$

$$a^x \times a^y \times a^z = (x+y+z)^y \times (x+y+z)^z \times (x+y+z)^x$$

$$\Rightarrow a^{x+y+z} = (x+y+z)^{(x+y+z)}$$

$$\therefore \textcircled{a = x+y+z}$$

11. If $a^x = (x+y+z)^y$, $a^y = (x+y+z)^z$ and $a^z = (x+y+z)^x$, then $x+y+z = ?$ ($a \neq 0$)

(a) 0

(b) 1

(c) a^3

(d) a

$$a^m = b^m$$

$$\therefore \textcircled{a=b}$$

$$x=y=z$$

11. If $a^x = (x + y + z)^y$, $a^y = (x + y + z)^z$ and $a^z = (x + y + z)^x$, then $x + y + z = ?$ ($a \neq 0$)

(a) 0

(b) 1

(c) a^3

(d) a

$\rightarrow 3x$

$$\rightarrow a^x = 3x^x$$

$$\therefore a = 3x$$

Special Concept:

$$a+b+c=0$$

~~Ø~~ 1, -1, 0

~~✓~~ 2, -1, -1

12. If $\overset{1}{a} + \overset{-1}{b} + \overset{0}{c} = 0$, then the value of

$$\frac{a^2 + b^2 + c^2}{a^2 - bc}$$

(a) 0

(b) 1

(c) 2

(d) 3



$$\rightarrow \frac{1+1+0}{1-0} = \frac{2}{1}$$

13. If $\overset{1}{a} + \overset{-1}{b} + \overset{0}{c} = 0$ then the value of $\frac{a^2 - bc}{b^2 - ca}$

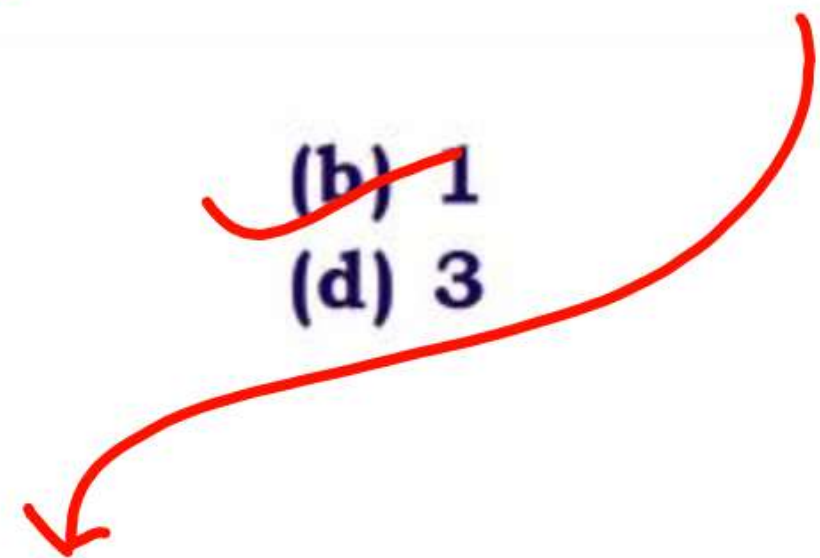
is :

(a) 0

(c) 2

~~(b) 1~~

(d) 3


$$= \frac{(1)^2 - 0}{(-1)^2 - 0} = \frac{1}{1}$$

14. If $\overset{1}{a} + \overset{-1}{b} + \overset{0}{c} = 0$ then the value of

$\frac{a^2 + b^2 + \cancel{c^2}}{ab + \cancel{bc} + \cancel{ca}}$ is

- (a) 2
(c) 0

- ✓ (b) -2
(d) 4


$$\frac{1+1}{-1} = -\frac{2}{1}$$

CONCEPT OF VALUE PUTTING

$$a+b=5$$

$$4 \quad 1$$

$$3 \quad 2$$

$$2 \quad 3$$

$$2 \vee 1 \in$$

$$\cancel{a+b=5}$$

$$(+)\ \cancel{a-b=3}$$

$$\cancel{a=84}$$

$$a=4$$

$$\therefore a+b=5$$

$$b=5-4=1$$

$$b=1$$

$$2 \vee 20 \neq$$

15. If $x + y = 1$, then what is the value of $x^3 + 3xy + y^3$?

SSC CGL MAINS (08/08/2022)

(a) 0

(b) 2

☒ (c) 1

(d) -1

$x + y = 1$
1, 0

$\rightarrow (1)^3 + 0 + 0$
 $= 1$

16. If $a^5 + b^1 = 6$ and $ab = 5$, then what is the value of $a^3 + b^3$?

SSC CGL MAINS (08/08/2022)

(a) 136

(c) 116

☒ (b) 126

(d) 106

$\rightarrow 125 + 1$
 $= 126$

5×1

17. If $a^3 + b^3 = 217$ and $a + b = 7$, then the value of ab is:

SSC CPO 24/11/2020 (Shift- 02)

(a) -6

(b) -1

(c) 7

(d) 6

$6 \times 1 = 6$

$$x + \frac{1}{x} = 2 \quad \therefore x = 1$$

$$3 + \frac{3}{\cancel{2}}$$

$$x + \frac{9}{x} = \underline{6}$$

$$x = 3$$

$$x + \frac{16}{x} = 8$$

$$x = 4$$

$$x + \frac{25}{x} = 10$$

$$x = 5$$

$$r = 8$$

$$\begin{aligned} & r^4 + \frac{1}{r^3} \\ &= 8^4 + \frac{1}{8^3} \\ &= 4096 + \frac{1}{512} \\ &= 4096 \frac{1}{512} \end{aligned}$$

✓ 18. If $r + \frac{64}{r} = 16$, then the value of $r^4 + \frac{1}{r^3}$ is.

SSC CHSL 25/05/2022 (Shift- 03)

✓ (a) $4096 \frac{1}{512}$

(b) 512

(c) $512 \frac{1}{4096}$

(d) 4096

$$2 + \frac{1}{3} = 2\frac{1}{3}$$

19. If $a - \frac{12}{a} = 1$, $a > 0$, find $a^2 + \frac{16}{a^2}$?

(a) 15

(b) 19

(c) 17

(d) 11

put $a=4$
 $4 - \frac{12}{4} = 1$
 $= 4 - 3$
 $= 1$

$a^2 + \frac{16}{a^2}$
 $= 16 + \frac{16}{16}$
 $= 17$

20. If $(x+6y) = 8$, and $xy = 2$, where $x > 0$, what is the value of $(x^3 + 216 y^3)$?

SSC CGL 11/04/2022 (Shift- 01)

(a) 288

(c) 368

☒ (b) 224

(d) 476

$$8 + 216 \times 1$$
$$= 224$$

21. If $a^2 + b^2 = 65$ and $ab = 8$, $a > b$ > 0 , then find the value of $a^2 - b^2$.

SSC CGL 21/04/2022 (Shift- 02)

(a) 65

(b) 60

(c) 72

✓ (d) 63

$$8^2 + 1^2 = \textcircled{65}$$

$$ab = 8$$

$$8 \times 1 = 8$$

$$\begin{aligned} \therefore a^2 - b^2 \\ &= 8^2 - 1^2 \\ &= 63 \end{aligned}$$

22. ✓
H.W If $x^2 + 9y^2 = 40$ and $xy = 4$, where $x > 0$, $y > 0$, then what is the value of $(x^3 + 27y^3)$?

SSC PHASE IX 2022

(a) 224

(b) 416

(c) 440

(d) 800

23. If $xy = -6$ and $x^3 + y^3 = 19$ (x and y are integers), then what is the value of

$$\frac{1}{x^{-1}} + \frac{1}{y^{-1}} ?$$

SSC CGL MAINS (08/08/2022)

- (a) -2
- (c) -1

- (b) 2
- (d) 1

$x + y$
 $= 3 - 2$
 $= 1$

24. If $a^2 + b^{-1} + c^{-1} = 0$, then

$\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b} \right) \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \right)$ is :

SSC CGL 06/06/2019 (Shift- 01)

(a) 8

(b) 9

(c) -3

(d) 0

$$\begin{aligned} & \rightarrow \left(\frac{2+(-1)}{(-1)} + \frac{(-1)+(-1)}{2} + \frac{(-1)+2}{-1} \right) \left(\frac{2}{-1+2} + \frac{-1}{2+(-1)} + \frac{-1}{(-1)+2} \right) \\ & = \left(\frac{-1}{-1} - \frac{1}{1} - \frac{1}{1} \right) \left(\frac{2}{1} - \frac{1}{1} - \frac{1}{1} \right) \\ & = (-3)(-3) \\ & = 9 \end{aligned}$$

$$x+z=y$$
$$1+1=2$$

25. If $x - y + z = 0$, then find the value of

$$\frac{y^2}{2xz} - \frac{x^2}{2yz} - \frac{z^2}{2xy}.$$

SSC CGL 12/04/2022 (Shift- 03)

$$\frac{4}{2 \times 1 \times 1} - \frac{1}{2 \times 2 \times 1} - \frac{1}{2 \times 1 \times 2} \quad (a) \quad \frac{3}{2}$$
$$= \frac{2}{1} - \frac{1}{4} - \frac{1}{4}$$
$$= \frac{8-1-1}{4} = \frac{6}{4} = \frac{3}{2}$$

(b) $\frac{1}{2}$

(c) -6

(d) $-\frac{3}{2}$

26. If $\overset{2}{a} + \overset{-1}{b} + \overset{-1}{c} = 0$, then what is the value of

$$\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab} ?$$

SSC CGL 18/08/2021 (Shift- 03)

(a) 1

(b) -3

(c) -1

☒ (d) 3


$$\begin{aligned} & \rightarrow \frac{(-2)^2}{1} + \frac{(-1)^2}{-2} + \frac{(1)^2}{(-2)} \\ &= \frac{4}{1} - \frac{1}{2} - \frac{1}{2} \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

verify

27. If $a + b - c = 0$, then what is the value of

$$\begin{matrix} a+b=c \\ 1 \quad 1 \quad 2 \end{matrix}$$

$$\frac{(b-c)^2}{4bc} + \frac{(c-a)^2}{4ca} - \frac{(a+b)^2}{4ab} ?$$

SSC CHSL 06/08/2021 (Shift- 01)

$$\begin{aligned} & \frac{(-1)^2}{4 \times 1 \times 2} + \frac{(1)^2}{4 \times 2 \times 1} - \frac{(2)^2}{4 \times 1 \times 1} \\ &= \frac{1}{8} + \frac{1}{8} - 1 \\ &= \frac{1}{4} - 1 \\ &= -\frac{3}{4} \end{aligned}$$

☒ (a) $-\frac{3}{4}$

(b) $\frac{3}{4}$

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

28. If $\overset{-2}{x} + \overset{1}{y} + \overset{1}{z} = 0$, then what will be the value

of $\left(\frac{x^2}{yz}\right) + \left(\frac{y^2}{zx}\right) + \left(\frac{z^2}{xy}\right)$?

SSC CHSL 24/05/2022 (Shift- 01)

$$\frac{4}{1} + \frac{1}{-2} + \frac{1}{-2}$$

$$= 4 - \frac{1}{2} - \frac{1}{2}$$

$$= 4 - 1$$

$$= 3$$

~~(a) $\frac{3x(4+1+1)}{-2}$~~

~~(c) $\frac{4 \times 1 \times 1}{-2}$~~

~~(b) $\overset{4+1+1=6}{x^2 + y^2 + z^2}$~~

✓ (d) 3

29. If $a + b + c = 0$, then

$$\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b} \right) \left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \right) \text{ is :}$$

SSC CGL 06/06/2019 (Shift- 01)

(a) 8

(c) - 3

 (b) 9

(d) 0

30. If $x = \frac{a}{b} + \frac{b}{a}$, $y = \frac{b}{c} + \frac{c}{b}$, $z = \frac{c}{a} + \frac{a}{c}$,
then what is the value of $xyz - x^2 - y^2 - z^2 = ?$

If $a=b=c=1$

$\therefore x = 1+1=2$

$x=y=z=2$

(a) -4

(c) -1

(b) -2

(d) -6

$8 - 4 - 4 - 4 = -4$

$$\frac{a^2}{a^2} = \frac{a^2}{a^2}$$

$$\boxed{a^2 = c}$$

1 1

$$abc = 180$$

$$1 \times b \times 1 = 180$$

$$\boxed{b = 180}$$

31. If $a^3b = abc = 180$, a, b, c are positive integers, then the value of c is :

(a) 110

(c) 4

✓ (b) 1

(d) 25

put $a=1$

$\therefore x=2$

$y=1-1=0$

32. If $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$ then

$\sqrt{x^4 + \cancel{y^4} - 2\cancel{x^2}y^2}$ is equal to :

SSC CGL 06/06 2019 (Shift- 01)

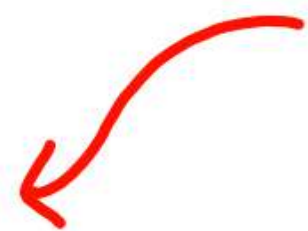
(a) $16a^2$ 16

(b) 8

(c) $\frac{8}{a^2}$ 8

☒ (d) 4

x^2
 $= 4$



33. If $x = 2$, then $x^3 + 6x + x^2$ is equal to :
 8

SSC CGL 07/06/2019 (Shift- 01)

(a) 12

(b) 6

☒ (c) 8

(d) 4

$$x = 2 - p$$

$$\Rightarrow x + p = 2$$

$$\Rightarrow (x + p)^3 = (2)^3$$

$$\Rightarrow x^3 + 3xp(x + p) + p^3 = 8$$

$$\Rightarrow x^3 + 3xp \times 2 + p^3 = 8$$

$$\Rightarrow x^3 + 6xp + p^3 = 8$$

33. If $x = 2 - p$, then $x^3 + 6xp + p^3$ is equal to :

SSC CGL 07/06/2019 (Shift- 01)

(a) 12

✓ (c) 8

(b) 6

(d) 4

34. Find the product of

$$(a + b + 2c)(a^2 + b^2 + 4c^2 - ab - 2bc - 2ca).$$

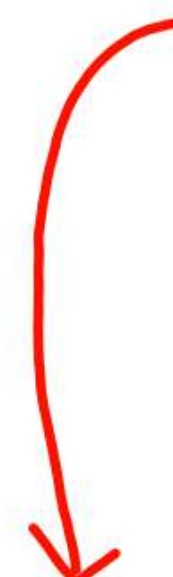
SSC CGL 07/03/2020 (Shift- 03)

☒ (a) $a^3 + b^3 + 8c^3 - 6abc$

(b) $a^3 + b^3 + 6c^3 - 6abc$

(c) $a^3 + b^3 + 8c^3 - 2abc$

(d) $a^3 + b^3 + 8c^3 - abc$


$$\begin{aligned} & a^3 + b^3 + 8c^3 - 3 \times a \times b \times 2c \\ &= \underline{a^3 + b^3 + 8c^3 - 6abc} \end{aligned}$$

35. If $a + b = 5$ and $ab = 10$,
then find the value of $a^2 + b^2$

SSC CGL 18/04/2022 (Shift- 01)

(a) 15
(c) 5

(b) 45
(d) 40

$$a+b=5 \quad ab=10$$

$$\therefore (a+b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow 25 = a^2 + b^2 + 20$$

$$\Rightarrow a^2 + b^2 = 5$$

36. If $a + b = 11$ and $ab = 28$,
then find the value of $a^3 + b^3$

SSC CGL 19/04/2022 (Shift- 03)

- ☒ (a) 407
(c) 2255

- (b) 1639
(d) 1093

$$a+b=11$$
$$7+4$$

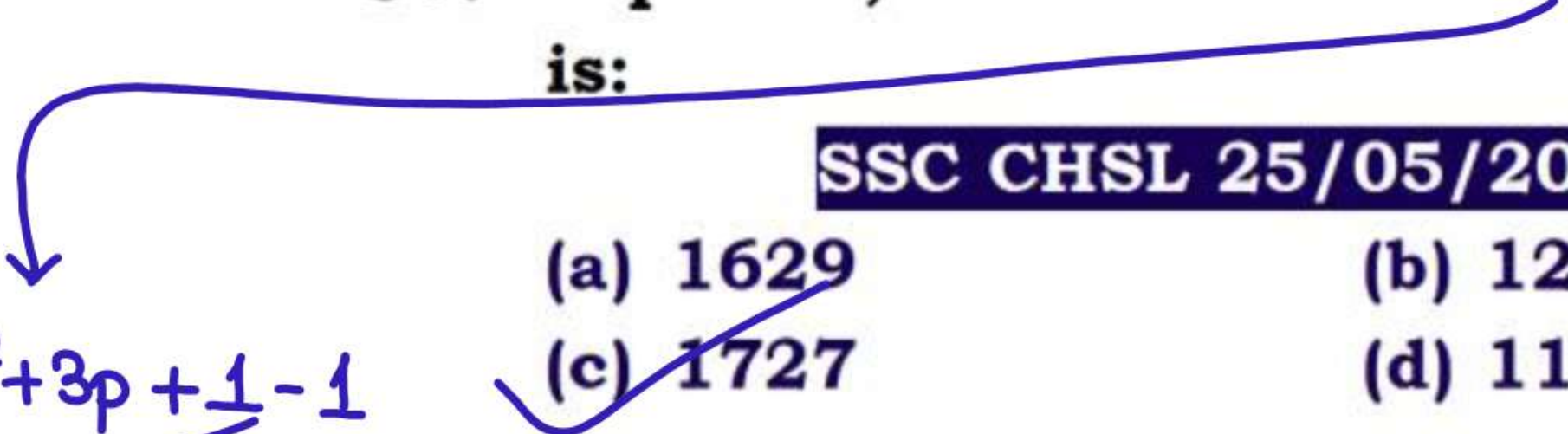
$$ab=28$$
$$7 \times 4$$

$$a^3+b^3$$
$$7^3+4^3$$
$$=343+64$$
$$=407$$

MISCELLANEOUS QUESTIONS

37. If $p = 11$, then the value of $p(p^2 + 3p + 3)$ is:

SSC CHSL 25/05/2022 (Shift- 03)


$$p^3 + 3p^2 + 3p + 1 - 1$$

$$= (p+1)^3 - 1$$

$$= (12)^3 - 1$$

$$= 1728 - 1$$

$$= 1727$$

(a) 1629

(c) 1727

(b) 1225

(d) 1111

38. *H.W* If $p = 38$, then the value of $p(p^2 + 3p + 3)$ is _____.

SSC CHSL 06/06/2022 (Shift- 03)

(a) 39313

(b) 59319

(c) 39318

(d) 59318

39. If $x = 9$, then the value of $x^5 - 10x^4 + 10x^3 - 10x^2 + 10x - 1$ is:

SSC CHSL 30/05/2022 (Shift- 02)

(a) 10

☒ (b) 8

(c) 9

(d) 1

$$\begin{aligned} & x^5 - 10x^4 + 10x^3 - 10x^2 + 10x - 1 \\ = & x^5 - 9x^4 - x^4 + 9x^3 + x^3 - 9x^2 - x^2 + 9x + x - 1 \\ = & x^5 - x \cdot x^4 - x^4 + x \cdot x^3 + x^3 - x \cdot x^2 - x^2 + x \cdot x + x - 1 \\ = & \cancel{x^5} - \cancel{x^5} - \cancel{x^4} + \cancel{x^4} + \cancel{x^3} - \cancel{x^3} - \cancel{x^2} + \cancel{x^2} + x - 1 \\ = & x - 1 \\ = & 9 - 1 \\ = & 8 \end{aligned}$$

* $x=9$

$$\underbrace{x^5 - 10x^4 + 10x^3 - 10x^2 + 10x - 1}_{+9x+x-1} = x-1 = 8 \checkmark$$

* $x=12$

$$\underbrace{x^6 - 13x^5 + 13x^4 - 13x^3 + 13x^2 - 13x + 5}_{-12x, -x+5} = -x+5 = -12+5 = -7$$

* $x=15$

$$\underbrace{x^5 - 16x^4 + 16x^3 - 16x^2 + 20x - 3}_{15x, +5x-3}$$

$$= 5x - 3$$

$$= 75 - 3$$

$$= \underline{\underline{72}}$$

$$(a^4 + a^2b^2 + b^4) = (a^2 + ab + b^2)(a^2 - ab + b^2)$$

$$243 = 13 \times 21$$

$$\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab} = \frac{3}{-4} = -\frac{3}{4}$$

40. If $a^4 + b^4 + a^2b^2 = 273$ and $a^2 + b^2 - ab = 21$, then one of the values of $\left(\frac{1}{a} + \frac{1}{b}\right)$ is:

SSC CGL 24/08/2021 (Shift- 01)

(a) $-\frac{9}{4}$

(b) $-\frac{3}{4}$

(c) $\frac{9}{8}$

(d) $\frac{3}{2}$

$$a^2 + b^2 = \frac{13+21}{2} = 17$$

$$ab = \frac{13-21}{2} = -4$$

$$\begin{aligned} \therefore (a+b)^2 &= a^2 + b^2 + 2ab \\ &= 17 - 8 \\ &= 9 \end{aligned}$$

$$\begin{aligned} (a+b)^2 &= 9 \\ a+b &= 3 \end{aligned}$$

$$(x^4 + x^2y^2 + y^4) = (x^2 + xy + y^2)(x^2 - xy + y^2) \quad 41. \text{ If } x^4 + y^4 + x^2y^2 = 17\frac{1}{16} \text{ and } x^2 - xy + y^2$$

$$\frac{273}{16} = \frac{13}{4} \times \frac{21}{4}$$

$$= 5\frac{1}{4}, \text{ then one of the values of } (x - y) \text{ is:}$$

SSC CGL MAINS 03/02/2022

$$(x - y)^2 = x^2 + y^2 - 2xy$$

$$(x - y)^2 = \left(\frac{\frac{13}{4} + \frac{21}{4}}{2} \right) - 2 \left(\frac{\frac{13}{4} - \frac{21}{4}}{2} \right)$$

$$= \frac{34}{8} - \frac{2 \times (-8)}{8} = \frac{34}{8} + \frac{16}{8} = \frac{50}{8} = \frac{25}{4}$$

$$\therefore (x - y)^2 = \frac{25}{4}$$

$$x - y = \frac{5}{2}$$

$$(a) \frac{5}{2}$$

$$(b) \frac{3}{4}$$

$$(c) \frac{5}{4}$$

$$(d) \frac{3}{2}$$

$$\frac{13}{4} \times \frac{21}{4} = 13 \times \frac{21}{4}$$

$$2^x = 3^y = 6^z = k$$

$$\Rightarrow \begin{array}{c|c|c} 2^x = k^1 & 3^y = k & 6^z = k \\ \hline \Rightarrow 2^1 = k^{\frac{1}{x}} & 3^1 = k^{\frac{1}{y}} & 6^1 = k^{\frac{1}{z}} \end{array}$$

$$\begin{aligned} 2 \times 3 &= 6 \\ \Rightarrow k^{\frac{1}{x}} \times k^{\frac{1}{y}} &= k^{\frac{1}{z}} \\ \Rightarrow k^{\frac{1}{x} + \frac{1}{y}} &= k^{\frac{1}{z}} \\ \Rightarrow \frac{1}{x} + \frac{1}{y} &= \frac{1}{z} \end{aligned}$$

42. If $2^x = 3^y = 6^z$, then $\frac{1}{x} + \frac{1}{y} - \frac{1}{z}$ is equal to:

SSC Phase X 01/08/2022 (Shift- 03)

(a) 1

(b) $\frac{3}{2}$

☒ (c) 0

(d) $-\frac{1}{2}$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} - \frac{1}{z} = 0$$

$$a^m \times a^n = a^{m+n}$$

$$a^m = a^n$$

$$3^a = 3^{3b} = 3^{4c}$$

$$a = 3b = 4c$$

$$a : b : c$$

$$12 : 4 : 3$$

$$12 \left(\frac{1}{12} + \frac{1}{8} + \frac{1}{15} \right)$$

$$= 12 \left(\frac{20+30+16}{240} \right)$$

$$= \frac{12 \times 66}{240} = \frac{33}{10}$$

43. If $3^a = 27^b = 81^c$ and $abc = 144$, then the

value of $12 \left(\frac{1}{a} + \frac{1}{2b} + \frac{1}{5c} \right)$ is :

SSC CGL 06/03/2020 (Shift- 01)

(a) $\frac{17}{120}$

(b) $\frac{18}{10}$

(c) $\frac{18}{120}$

(d) $\frac{33}{10}$

$$\therefore 2x + 3y + 5z = 10 + 24 + 10 = 44$$

44. If $\underline{2}^{x+y-2z} = \underline{8}^{8z-5-y}$; $5^{4y-6z} = 25^{y+z}$; $3^{4x-3z} = 9^{x+z}$ then the value of $2x + 3y + 5z$ is :

CHSL 13/10/2020 (Shift- 01)

(a) 56

(b) 44

(c) 32

(d) 28

$$2^{x+y-2z} = 2^3(8z-5-y)$$

$$x+y-2z = 24z-15-3y$$

$$\boxed{x+4y-26z=15}$$

$$\frac{5}{2}z + 16z - 26z = -15$$

$$\Rightarrow \frac{5}{2}z - 10z = -15$$

$$\Rightarrow \frac{-15}{2}z = -15$$

$$\boxed{z=2}$$

$$5^{4y-6z} = 5^2(y+z)$$

$$4y-6z = 2y+2z$$

$$\boxed{2y = 8z}$$

$$y = 8z$$

$$\boxed{y=8}$$

$$3^{4x-3z} = 3^{2(x+z)}$$

$$4x-3z = 2x+2z$$

$$2x = 5z$$

$$x = \frac{5}{2}z$$

$$\boxed{x = \frac{5}{2} \times 2}$$

45. If $5^{\sqrt[3]{x}} + 12^{\sqrt[3]{x}} = 13^{\sqrt[3]{x}}$, then the value of x is :

SSC CHSL 05/07/2019 (Shift- 02)

(a) 2

(c) 1

☒ (b) 8

(d) 4

$$5^{\sqrt[3]{x}} + 12^{\sqrt[3]{x}} = 13^{\sqrt[3]{x}}$$

$$5^2 + 12^2 = 13^2$$

$$\sqrt[3]{x} = 2$$

$$\therefore (x^{\frac{1}{3}})^3 = (2)^3$$

$$x = 8$$

46. If $6^{\sqrt[4]{x}} + 8^{\sqrt[4]{x}} = 10^{\sqrt[4]{x}}$, then the value of x is :

SSC CHSL 08/07/2019 (Shift- 02)

(a) 2

(c) 4

☒ (b) 16

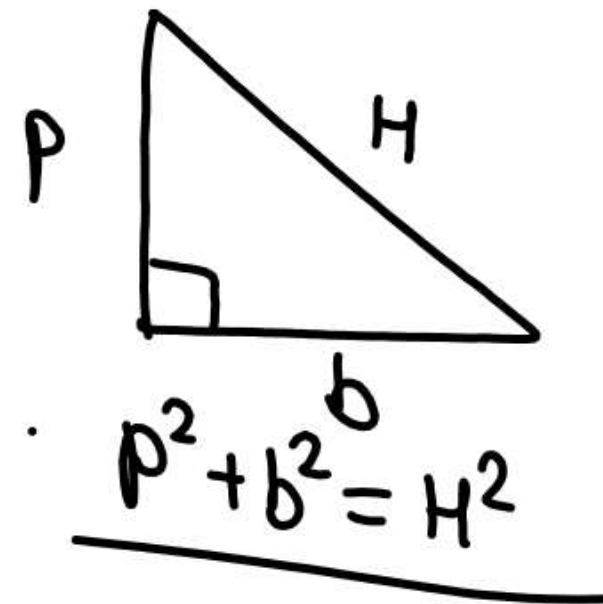
(d) 8

$$6^{\sqrt[4]{x}} + 8^{\sqrt[4]{x}} = 10^{\sqrt[4]{x}}$$
$$6^2 + 8^2 = 10^2$$

$$\sqrt[4]{x} = 2$$

$$(x^{\frac{1}{4}})^4 = (2)^4$$

$$x = 16$$



Special Type

$$* \sqrt{8 + 2\sqrt{15}} = \sqrt{5} + \sqrt{3}$$

$$* \sqrt{14 + 2\sqrt{45}} = \sqrt{9} + \sqrt{5} = 3 + \sqrt{5}$$

$$* \sqrt{8 - 2\sqrt{12}} = \sqrt{6} - \sqrt{2}$$

$$\begin{aligned} & \sqrt{86 - 60\sqrt{2}} \\ &= \sqrt{86 - 2 \times 30\sqrt{2}} \\ &= \sqrt{86 - 2\sqrt{900 \times 2}} \\ &= \sqrt{86 - 2\sqrt{1800}} \\ &= \sqrt{50} - \sqrt{36} \\ &\Rightarrow 5\sqrt{2} - 6 \end{aligned}$$

$$5\sqrt{2} - 6 = a - b\sqrt{2}$$

$$\begin{array}{l|l} 5\sqrt{2} = -b\sqrt{2} & a = -6 \\ \hline b = -5 & \end{array}$$

47. If $\sqrt{86 - 60\sqrt{2}} = a - b\sqrt{2}$, then what will be the value of $\sqrt{a^2 + b^2}$, correct to one decimal place?

CGL Tier-II 11/09/2019

(a) 8.4

(b) 8.2

☒ (c) 7.8

(d) 7.2

$$\sqrt{a^2 + b^2} = \sqrt{36 + 25} = \sqrt{61} = \underline{\underline{7.8}}$$

$$\begin{aligned}
x &= \sqrt{1 + \frac{\sqrt{3}}{2}} - \sqrt{1 - \frac{\sqrt{3}}{2}} \\
&= \sqrt{\frac{2 + \sqrt{3}}{2}} - \sqrt{\frac{2 - \sqrt{3}}{2}} \\
&= \sqrt{\frac{4 + 2\sqrt{3}}{4}} - \sqrt{\frac{4 - 2\sqrt{3}}{4}} \\
&= \left(\frac{\sqrt{3} + \sqrt{1}}{2} \right) - \frac{(\sqrt{3} - \sqrt{1})}{2} \\
&= \frac{\cancel{\sqrt{3}} + 1 - \cancel{\sqrt{3}} + 1}{2} \\
&= \frac{2}{2} = 1
\end{aligned}$$

48. If $x = \sqrt{1 + \frac{\sqrt{3}}{2}} - \sqrt{1 - \frac{\sqrt{3}}{2}}$, then the value of

$\frac{\sqrt{2} - x}{\sqrt{2} + x}$ will be closest to :
 $\left(\frac{\sqrt{2}-1}{\sqrt{2}+1} \right) \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$

CGL Tier-II 11/09/2019

$$= \frac{(\sqrt{2}-1)^2}{(2-1)} = \frac{2+1-2\sqrt{2}}{1}$$

$$\begin{aligned} &= 3 - 2\sqrt{2} \\ &= 3 - 2 \times 1.414 \\ &= 3 - 2.828 \\ &= 0.17 \end{aligned}$$

(a) 0.17
(c) 1.4

(b) 0.12
(d) 1.2

$$\begin{aligned} \sqrt{2} &= 1.414 \\ \sqrt{3} &= 1.732 \end{aligned}$$

48. If $x = \sqrt{1 + \frac{\sqrt{3}}{2}} - \sqrt{1 - \frac{\sqrt{3}}{2}}$, then the value of

$\frac{\sqrt{2} - x}{\sqrt{2} + x}$ will be closest to :

CGL Tier-II 11/09/2019

$$= \frac{1.414 - 1}{1.414 + 1} = \frac{0.414}{2.414} = 0.17$$

(a) 0.17

(c) 1.4

(b) 0.12

(d) 1.2

$$\sqrt{2} = 1.414$$
$$\sqrt{3} = 1.732$$

$$\begin{array}{r} 0.17 \\ 2414 \overline{) 4140} \\ \underline{2414} \\ 17260 \\ \underline{-16898} \\ 362 \end{array}$$

Q. If $x = \sqrt{1 + \frac{\sqrt{3}}{2}} - \sqrt{1 - \frac{\sqrt{3}}{2}}$, then the value of

$\frac{\sqrt{3} - x}{\sqrt{3} + x}$ will be closest to :

CGL Tier-II 11/09/2019


$$\rightarrow \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{1.732 - 1}{1.732 + 1} = \frac{\cancel{0.732}}{\cancel{2.732}} \frac{183}{683} = 0.27$$

$a=4$ $b=3$ $c=1$

49. $ab(a-b) + bc(b-c) + ca(c-a)$ is equal to :

CGL Tier-II 13 09/2019

$$= \cancel{12 \times 1} + 3 \times 2 + 4 \times \cancel{(-3)}$$

= ⑥

~~(a)~~ $(a + b)(b - c)(c - a)$
~~(b)~~ $(a - b)(b + c)(c - a)$
~~(c)~~ $(a - b)(b^2 - c)(c - a)$
✓ (d) $(b - a)(b - c)(c - a)$
 $(-1)(2)(-3)$
= ⑥

$$\begin{aligned}
 x &= (3^{\cancel{3}})^{\frac{1}{\cancel{6}2}} - \left(\frac{27}{4}\right)^{\frac{1}{2}} \\
 &= \sqrt{3} - \frac{3\sqrt{3}}{2} \\
 &= -\frac{\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 & x^2 + y^2 \\
 &= \frac{3}{4} + \frac{49}{9} \\
 &= \frac{27 + 196}{36} \\
 &= \frac{223}{36}
 \end{aligned}$$

50. Let $x = \sqrt[6]{27} - \sqrt{6\frac{3}{4}}$ and $y = \frac{\sqrt{9} + \sqrt{121} + \sqrt{49}}{\sqrt{45} + \sqrt{605} + \sqrt{245}}$, then the value of $x^2 + y^2$ is:

CGL Tier-II 13/09/2019

☒ (a) $\frac{223}{36}$

(b) $\frac{221}{36}$

(c) $\frac{221}{9}$

(d) $\frac{227}{9}$

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\sqrt[4]{x} = x^{\frac{1}{4}}$$

Concept

$$(a^4 + a^2b^2 + b^4) = (a^2 + ab + b^2)(a^2 - ab + b^2)$$

$$91 = 13 \times 7$$

$$* \quad a^2 + b^2 = \frac{13+7}{2} = 10$$

$$* \quad ab = \frac{13-7}{2} = 3$$

Concept

$$(a^4 + a^2b^2 + b^4) = (a^2 + ab + b^2)(a^2 - ab + b^2)$$

$$91 = 13 \times 7$$

$$\begin{array}{r} a^2 + \cancel{ab} + b^2 = 13 \\ + \quad a^2 - \cancel{ab} + b^2 = 7 \\ \hline 2a^2 + 2b^2 = (13+7) \\ a^2 + b^2 = \frac{(13+7)}{2} \end{array}$$

$$\begin{array}{r} \cancel{a^2} + ab + \cancel{b^2} = 13 \\ - \quad \cancel{a^2} + ab + \cancel{b^2} = 7 \\ \hline 2ab = 13-7 \\ ab = \frac{13-7}{2} \end{array}$$

51. If $x^4 + x^2y^2 + y^4 = 21$ and $x^2 + xy + y^2 = 7$,

$(x^4 + x^2y^2 + y^4) = (x^2 + xy + y^2)(x^2 - xy + y^2)$ then the value of $\left(\frac{1}{x^2} + \frac{1}{y^2}\right)$ is :

SSC CGL 03/03/2020 (Shift- 02)

Q1 7 x 3

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{y^2 + x^2}{(xy)^2}$$
$$= \frac{5}{(2)^2} = \frac{5}{4}$$

(a) $\frac{5}{2}$

(b) $\frac{7}{4}$

✓ (c) $\frac{5}{4}$

(d) $\frac{7}{3}$

$$x^2 + y^2 = \frac{10}{2} = 5$$

$$xy = \frac{4}{2} = 2$$

52. If $x^4 + x^2y^2 + y^4 = 273$ and $x^2 - xy + y^2 = 13$, then the value of xy is :

SSC CGL 05/03/2020 (Shift- 02)

☒ (a) 4
☐ (c) 10

☐ (b) 8
☐ (d) 6

$$(x^4 + x^2y^2 + y^4) = (x^2 + xy + y^2)(x^2 - xy + y^2)$$
$$273 = 21 \times 13$$

$$xy = \frac{21 - 13}{2} = 4$$

$$4a^2 + ab^2 = \frac{7+13}{2} = 10$$

$$6ab = \frac{7-13}{2} = -3$$

$$\rightarrow 3ab = -\frac{3}{2}$$

53. If $16a^4 + 36a^2b^2 + 81b^4 = 91$ and $4a^2 + 9b^2 - 6ab = 13$, then what is the value of $3ab$?

SSC CGL 04/03/2020 (Shift- 01)

(a) -3

(b) $\frac{3}{2}$

(c) 5

(d) $-\frac{3}{2}$

$$\underset{91}{(16a^4 + 36a^2b^2 + 81b^4)} = \underset{7}{(\underbrace{4a^2 + 9b^2}_{7} + 6ab)} \underset{13}{\times (4a^2 + 9b^2 - 6ab)}$$

54. If $\frac{7(x^2 + 1) - 17x}{7x} = 2$, then $\left(\frac{x+1}{\sqrt{x}}\right)^2 = ?$

☒ (a) 6.45

(b) 6.53

(c) 6.86

(d) 6.31

$$x + \frac{1}{x} - \frac{17}{7} = 2$$

$$\Rightarrow x + \frac{1}{x} = 2 + \frac{17}{7}$$

$$\Rightarrow x + \frac{1}{x} = \frac{31}{7}$$

$$\frac{x^2 + 1 + 2x}{x}$$

$$= x + \frac{1}{x} + 2$$

$$= \frac{31}{7} + 2$$

$$= \frac{45}{7} = 6.45$$

$$14\sqrt{2} - 7x = 7x + \frac{14}{x}$$

$$\Rightarrow \cancel{14\sqrt{2}} = \cancel{14x} + \cancel{\frac{14}{x}}$$

$$\therefore x + \frac{1}{x} = \sqrt{2}$$

$$\rightarrow x^2 + \frac{1}{x^2} = 2 - 2 = 0$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 0$$

$$\Rightarrow \frac{x^4 + 1}{x^2} = 0$$

$$\Rightarrow x^4 + 1 = 0$$

$$\Rightarrow \boxed{x^4 = -1}$$

55. If $\frac{7(2\sqrt{2} - x)}{x} = 7x + \frac{14}{x^2}$ then

$$\sqrt{(23x^{64} + 42x^{12} + 163)}$$

(a) 12

(b) $12\sqrt{2}$

(c) $12\sqrt{3}$

(d) 24

$$\begin{aligned} & \sqrt{23(x^4)^{16} + 42(x^4)^3 + 163} \\ &= \sqrt{23(-1)^{16} + 42(-1)^3 + 163} \\ &= \sqrt{23 - 42 + 163} \\ &= \sqrt{144} = 12 \end{aligned}$$

$$k^3 = x^1$$

$$8x^2 + 15x - 2 = 0$$

$$\Rightarrow 8x^2 + 16x - x - 2 = 0$$

$$\Rightarrow 8x(x+2) - 1(x+2) = 0$$

$$(x+2)(8x-1) = 0$$

$$(x = -2) \text{ or } 8x = 1$$

$$x = \frac{1}{8}$$

$$\therefore k^3 = \frac{1}{8}$$

$$\therefore k = \frac{1}{2}$$

56. If $8k^6 + 15k^3 - 2 = 0$, then the positive

value of $\left(k + \frac{1}{k}\right)$ is:

SSC CGL 12/04/2022 (Shift- 03)

✓ (a) $2\frac{1}{2}$

(b) $2\frac{1}{8}$

(c) $8\frac{1}{2}$

(d) $8\frac{1}{8}$

ans = $\frac{1}{2} + 2 = 2\frac{1}{2}$

57. If $x = \sqrt{1 + \frac{\sqrt{3}}{2}} - \sqrt{1 - \frac{\sqrt{3}}{2}}$ then the value of

$\frac{\sqrt{3} - x}{\sqrt{3} + x}$ (correct to one decimal place) is?

SSC CGL MAINS 29/01/2022

(a) 0.25

(b) 0.17

(c) 0.19

✓ (d) 0.27

58. If $(ab + bc + ca) = 0$, then what is the value

of $\left(\frac{1}{a^2 - bc} + \frac{1}{b^2 - ca} + \frac{1}{c^2 - ab} \right)$?

SSC CHSL 09/06/2022 (Shift- 01)

(a) 2

(c) 1

☒ (b) 0

(d) $a + b + c$

✓ $ab + ac = -bc$

✓ $ab + bc = -ca$

✓ $bc + ca = -ab$

$$\begin{aligned} & \frac{1}{a^2 + ab + ac} + \frac{1}{b^2 + ab + bc} + \frac{1}{c^2 + bc + ac} \\ &= \frac{1}{a(a+b+c)} + \frac{1}{b(b+a+c)} + \frac{1}{c(c+b+a)} \\ &= \frac{1}{a+b+c} \left[\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right] = \frac{1}{a+b+c} \left[\frac{bc + ac + ab}{abc} \right] = 0 \end{aligned}$$

~~59.~~ Quadratic If $px^3 + x^2 + 3x + q$ is exactly divisible by $(x + 2)$ and $(x - 2)$, then the values of p and q are:

SSC CHSL 08/06/2022 (Shift- 03)

(a) $p = -\frac{3}{4}$ and $q = 4$

(b) $p = \frac{3}{4}$ and $q = 4$

(c) $p = \frac{3}{4}$ and $q = -4$

(d) $p = -\frac{3}{4}$ and $q = -4$

put $x=1$

$$\frac{4 \times 9 \times 1}{6^2}$$

60. What is the value of the following expression?

$$2^2 \left(\frac{x^a}{x^b} \right)^{(a+b)} \times 3^2 \left(\frac{x^b}{x^c} \right)^{(b+c)} \times 6^{-2} \left(\frac{x^c}{x^a} \right)^{(a+c)}$$

SSC CHSL 08/06/2022 (Shift- 02)

- ☒ (a) 1
(c) 4

- (b) 0
(d) 9

60. What is the value of the following expression?

$$2^2 \left(\frac{x^a}{x^b} \right)^{(a+b)} \times 3^2 \left(\frac{x^b}{x^c} \right)^{(b+c)} \times 6^{-2} \left(\frac{x^c}{x^a} \right)^{(a+c)}$$

SSC CHSL 08/06/2022 (Shift- 02)

(a) 1

(b) 0

(c) 4

(d) 9

$$4(x^{a-b})^{a+b} \times 9(x^{b-c})^{b+c} \times \frac{1}{6^2}(x^{c-a})^{c+a}$$

$$= \cancel{4}(x)^{(a-b)(a+b)} \times \cancel{9}(x)^{(b-c)(b+c)} \times \frac{1}{36}(x)^{(c-a)(c+a)}$$

$$= x^{(a^2-b^2)} \times x^{(b^2-c^2)} \times x^{(c^2-a^2)}$$

$$= x^{\cancel{a^2-b^2} + \cancel{b^2-c^2} + \cancel{c^2-a^2}} = x^0 = 1$$

$$* \quad x^3 + y^3 + z^3 = 3xyz \quad x+y+z=0$$

$$* \quad (a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3 = 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$$

$$* \quad (a^7 - b^7)^3 + (b^7 - c^7)^3 + (c^7 - a^7)^3 = 3(a^7 - b^7)(b^7 - c^7)(c^7 - a^7)$$

$$(a^2 - b^2) = (a - b)(a + b)$$

$$(a^4 - b^4) = (a^2 - b^2)(a^2 + b^2)$$

$$(a^6 - b^6) = (a^3 + b^3)(a^3 - b^3)$$

$$(a^8 - b^8) = (a^4 - b^4)(a^4 + b^4)$$

61.

Simplify the following expression.

$$\frac{5(a^6 - b^6)^3 + 5(b^6 - c^6)^3 + 5(c^6 - a^6)^3}{2(a^3 - b^3)^3 + 2(b^3 - c^3)^3 + 2(c^3 - a^3)^3}$$

SSC CGL 24/08/2021 (Shift- 02)

$$\begin{aligned}
 &= \frac{5 \{ (a^6 - b^6)^3 + (b^6 - c^6)^3 + (c^6 - a^6)^3 \}}{2 \{ (a^3 - b^3)^3 + (b^3 - c^3)^3 + (c^3 - a^3)^3 \}} \\
 &= \frac{5 \times \cancel{3} (a^6 - b^6)(b^6 - c^6)(c^6 - a^6)}{2 \times \cancel{3} (a^3 - b^3)(b^3 - c^3)(c^3 - a^3)} \\
 &= \frac{5}{2} (a^3 + b^3)(b^3 + c^3)(c^3 + a^3)
 \end{aligned}$$

✓ (a) $\frac{5}{2} (a^3 + b^3)(b^3 + c^3)(c^3 + a^3)$

(b) $\frac{5}{2} (a^3 + b^3)(b^3 - c^3)(c^3 - a^3)$

(c) $\frac{5}{2} (a^3 - b^3)(b^3 + c^3)(c^3 + a^3)$

(d) $\frac{5}{2} (a^3 - b^3)(b^3 - c^3)(c^3 + a^3)$

62. If x^3 + $4x = 8$, find x^7 + $64x^2$?

(a) 32

(b) 64

(c) 128

(d) 256

$$x^3 + 4x = 8$$

$$\Rightarrow (x^3)^2 = (8 - 4x)^2$$

$$\Rightarrow x^6 = 64 + 16x^2 - 2 \times 8 \times 4x$$

$$\Rightarrow x^6 = 64 + 16x^2 - 64x$$

$$\Rightarrow x^7 = \underline{64x + 16x^3} - 64x^2$$

$$\Rightarrow \underline{x^7 + 64x^2} = 16(x^3 + 4x)$$

$$\Rightarrow x^7 + 64x^2 = 16 \times 8 = \boxed{128}$$

63. If $x^2 - 5x + 25 = 0$ then $x^5 - 5x^4 - 3100$?

(a) 25

(b) $25\sqrt{2}$

(c) $25\sqrt{3}$

(d) None

$$\begin{aligned} (x+5)(x^2-5x+25) &= 0 \\ \Rightarrow (x^3+5^3) &= 0 \\ \Rightarrow x^3 &= -125 \end{aligned}$$

$$\begin{aligned} &x^5 - 5x^4 - 3100 \\ &= x^3(x^2 - 5x) - 3100 \\ &= (-125)(-5) - 3100 \\ &= + 625 - 3100 \\ &= -2475 \end{aligned}$$

$$\frac{x(x-1)(x+5)(x+6) + 9}{x(x-1)} = 0$$

$$\Rightarrow x(x+5)(x-1)(x+6) + 9 = 0$$

$$\Rightarrow (x^2 + 5x)(x^2 + 6x - x - 6) + 9 = 0$$

$$\Rightarrow \underbrace{(x^2 + 5x)}_t \underbrace{(x^2 + 5x - 6)}_t + 9 = 0$$

$$\Rightarrow t(t-6) + 9 = 0$$

$$\Rightarrow t^2 - 6t + 9 = 0$$

$$\Rightarrow (t-3)^2 = 0$$

$$\Rightarrow \boxed{t=3}$$

$$t = x^2 + 5x = 3$$

64. If $\frac{(x+5)(x+6)}{x(x-1)} + \frac{9}{x(x-1)} = 0$, Find $3x^2 + 15x - 1$?

(a) 4

(b) 6

☒ (c) 8

(d) 10

$$\begin{aligned} & 3x^2 + 15x - 1 \\ &= 3(x^2 + 5x) - 1 \\ &= 3 \times 3 - 1 \\ &= 8 \end{aligned}$$

$$Q. \quad ax+by = p$$

$$ay-bx = q$$

$$\left. \begin{array}{l} ax+by = p \\ ay-bx = q \end{array} \right\} (a^2+b^2)(x^2+y^2) = (p^2+q^2)$$

$$\underline{\text{Sol}^n} \quad (ax+by)^2 = (p)^2$$

$$+ (ay-bx)^2 = (q)^2$$

$$\Rightarrow \underline{a^2x^2} + \underline{b^2y^2} + \cancel{2abxy} + \underline{a^2y^2} + \underline{b^2x^2} - \cancel{2abxy} = p^2 + q^2$$

$$\Rightarrow a^2(x^2+y^2) + b^2(y^2+x^2) = p^2 + q^2$$

$$\Rightarrow (x^2+y^2)(a^2+b^2) = p^2 + q^2$$

$$ax + by = 8$$

$$ay - bx = 6$$

$$(a^2 + b^2)(x^2 + y^2) = 8^2 + 6^2 = \underline{\underline{100}} \quad \checkmark$$

$$m \times n = 100 \quad \checkmark$$

$$m + n = 29 \quad \checkmark$$

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$$\therefore \frac{a^2 + b^2}{x^2 + y^2} = \frac{m}{n} = \frac{25}{4}$$

65. If $ax + by = 8$, $ay - bx = 6$, $\underline{a^2 + b^2} + x^2 + y^2 = 29$

Find $\frac{a^2 + b^2}{(x^2 + y^2)}$; $\underline{a^2 + b^2} > \underline{x^2 + y^2}$?

☒ (a) $\frac{25}{4}$

(b) $\frac{27}{4}$

(c) $\frac{23}{4}$

(d) $\frac{29}{4}$

66. If 3^a = $27^b = 81^c$ and $abc = 144$, then the value

of $12\left(\frac{1}{a} + \frac{1}{2b} + \frac{1}{5c}\right)$ is :

☒ (a) $\frac{33}{10}$

(b) $\frac{18}{10}$

(c) $\frac{17}{120}$

(d) $\frac{18}{120}$

67. If $\frac{a^2 + b^2 + c^2 - 1024}{ab - bc - ca} = -2$ and $a + b = 5c$,
where $c > 0$, then the value of c is _____.

(a) 8

(b) 4

(c) 12

(d) 5

$$\Rightarrow a^2 + b^2 + c^2 - 1024 = -2(ab - bc - ac)$$

$$\Rightarrow a^2 + b^2 + c^2 + 2(ab - bc - ac) = 1024$$

$$\Rightarrow (a + b - c)^2 = 1024$$

$$\Rightarrow a + b - c = \pm 32$$

$$\therefore a + b - c = 32$$

$$\Rightarrow \cancel{c = 32} \quad 8$$

68. If $a^3 - b^3 = 602$ and $a - b = 2$, then find the value of $(a^2 + b^2)$.

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(a) 156

(b) 240

(c) 202

(d) 260

$$\rightarrow 121 + 81 = 202$$

68. If $a^3 - b^3 = 602$ and $(a - b)^2 = (2)^2$ then find the value of $(a^2 + b^2)$.

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(a) 156

(b) 240

(c) 202

(d) 260

$(a-b)(a^2+b^2+ab) = 602$

$2(a^2+b^2+ab) = 602$

$\Rightarrow 2a^2 + 2b^2 + 2ab = 602$

$+ a^2 + b^2 - 2ab = 4$

$3a^2 + 3b^2 = 606$

$a^2 + b^2 = 202$

$$x-7 = 7^{\frac{2}{3}} + 7^{\frac{1}{3}}$$

$$\Rightarrow (x-7)^3 = (7^{\frac{2}{3}} + 7^{\frac{1}{3}})^3$$

$$7^{\frac{2}{3} + \frac{1}{3}} = 7^{\frac{3}{3}} = 7$$

69. If $x = 7 + 7^{\frac{2}{3}} + 7^{\frac{1}{3}}$, then which of the options below is correct.

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(a) $x^3 + 21x^2 - 126x - 252 = 0$

(b) $x^3 + 21x^2 - 126x + 252 = 0$

(c) $x^3 - 21x^2 - 126x + 252 = 0$

(d) $x^3 - 21x^2 + 126x - 252 = 0$

$$\Rightarrow x^3 - 3x \cdot x \cdot 7 + (x-7)^3 = (7^{\frac{2}{3}})^3 + 3 \cdot 7^{\frac{2}{3}} \cdot 7^{\frac{1}{3}} \cdot (7^{\frac{2}{3}} + 7^{\frac{1}{3}}) + (7^{\frac{1}{3}})^3$$

$$\Rightarrow x^3 - 21x^2 + 147x - 343 = 49 + 3 \cdot 7 \cdot (x-7) + 7$$

$$\Rightarrow x^3 - 21x^2 + 147x - 343 = 49 + 21x - 147 + 7$$

$$\Rightarrow x^3 - 21x^2 + 126x - 252 = 0$$

70. Select the option that is true regarding the following labelled Assertion (A) and Reason (R).

Assertion (A): The value of

$$\frac{[(3.9)^3 + 9 \times 1.3 \times 4.29 + 11.7 \times (1.1)^2 + 1.331]}{[1.23 \times 1.23 + 0.77(0.77 + 0.6 \times 4.1)]}$$

Handwritten annotations: (3.9) is circled and labeled 'a'; 9 is labeled '3(a^2)b'; 1.3 is labeled '3a(b)^2'; 4.29 is labeled '(1.1)^3'; 1.23 is labeled 'x'; 0.77 is labeled 'y'.

is 31.25.

Reason (R): Using $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ and $(x + y)^2 = x^2 + y^2 + 2xy$

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~~(a)~~ A is true and R is false.

~~(b)~~ Both A and R are true but R is not a correct explanation of A.

☒ (c) Both A and R are true and R is a correct explanation of A.

(d) A is false and R is true.

$$\frac{(3.9 + 1.1)^3}{(1.23 + 0.77)^2} = \frac{(5)^3}{(2)^2} = \frac{125}{4} = 31.25$$

Handwritten red arrow points from the handwritten expression to the Assertion (A) expression.