

NUMBER SYSTEM



**NO. OF ZERO AT THE
END OF AN EXPRESSION**

CLASS NOTES

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! or L

Meaning of Factorial

The product of first n consecutive natural numbers is called as $n!$.

प्रथम n क्रमागत प्राकृतिक संख्याओं का गुणनफल $n!$ कहलाता है।

E.g., $3!$ is defined as product of first three natural numbers.

$3!$ को पहले तीन प्राकृतिक संख्याओं के गुणनफल के रूप में परिभाषित किया गया है

$$3! = 1 \times 2 \times 3$$

$$3! = 6$$

$$4! = 1 \times 2 \times 3 \times 4 \\ = 24$$

$$1! = 1$$

$$2! = 2$$

$$3! = 6$$

$$4! = 24$$

$$5! = 120$$

$$6! = 720$$

$$7! = 5040$$

$$8! = 40320$$

Note :-

$$0! = 1$$

Factorial (! or L)

$$* 0! = 1$$

$$* 1! = 1$$

$$* 2! = 1 \times 2 = 2$$

$$* 3! = 1 \times 2 \times 3 = 6$$

$$* 4! = 1 \times 2 \times 3 \times 4 = 24$$

$$* 5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$$

$$* 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

$$* 7!! = 7 \times 5 \times 3 \times 1 = 105$$

$$* 7!!! = 7 \times 4 \times 1 = \underline{\underline{28}}$$

$$\begin{aligned} * 5!! \\ &= 5 \times 3 \times 1 \\ &= \underline{\underline{15}} \end{aligned}$$

$$\begin{aligned} (5!)! \\ &= (5 \times 4 \times 3 \times 2 \times 1)! \\ &= \underline{\underline{120!}} \end{aligned}$$

How to Calculate Number of zeros at the end of N

$$10 = 2^1 \times 5^1$$

$$100 = 2^2 \times 5^2$$

While calculating number of trailing zeroes at the end of any given number or expression we have to evaluate powers of 2 and 5.

किसी दी गई संख्या या व्यंजक के अंत में अनगामी शून्यों की संख्या की गणना करते समय हमें 2 और 5 की घातों की संख्या ज्ञात करना होता है।

$$A = 2^{10} \times 5^2$$

$$N.O.Z = \underline{\underline{2}}$$

$$B = 2^3 \times 5^{10}$$

$$N.O.Z = 3$$

$$C = 2^4 \times 5^3$$

$$N.O.Z = \underline{\underline{3}}$$

1.

The number of zeroes at the end of $21 \times 35 \times 625 \times 8 \times 165$ is :

21 $\times$ 35 $\times$ 625 $\times$ 8 $\times$ 165 के अंत में शून्यों की संख्या है:

(a) 1

☒ (b) 3

(c) 5

(d) 7

$$\begin{aligned} & \rightarrow 3 \times 7 \times 5 \times 5^4 \times 2^3 \times 5 \times 3 \times 11 \\ & = 2^3 \times 3^2 \times 5^6 \times 7^2 \times 11 \end{aligned}$$

$$N.O.Z = \underline{\underline{3}}$$

1. The number of zeroes at the end of $21 \times 35 \times 625 \times 8 \times 165$ is :

$21 \times \underline{35} \times \underline{625} \times \textcircled{8} \times \underline{165}$ के अंत में शून्यों की संख्या है:

(a) 1

(c) 5

☒ (b) 3

(d) 7

$$\text{Ans} = 8 = 2^3 =$$

2. The number of zeroes at the end of $25 \times 35 \times 40 \times 50 \times 60 \times 65$ is :

$25 \times 35 \times 40 \times 50 \times 60 \times 65$ के अंत में शून्यों की संख्या है:

☒ (a) 6
(c) 5

(b) 8
(d) 7

$\rightarrow 5^2 \times 5^1 \times 7 \times 2^3 \times 5^1 \times 5^2 \times 2^1 \times 3 \times 5^1 \times 2^2 \times 5^1 \times 13$

$2^6 \times 5^8 \times \dots$
N.O.Z = 6

Q. $1 \times 3 \times 5 \times 7 \times 9 \times 11$

N.O.Z = ?

ans $2 \rightarrow 0$ $5 \rightarrow 1$

N.O.Z = 0 //

3. The number of zeroes at the end of $1003 \times 1001 \times 999 \times \dots \times 123$ is :
- $1003 \times 1001 \times 999 \times \dots \times 123$ के अंत में शून्यों की संख्या है:
- (a) 224 (b) 217
- (c) 0 (d) 212

$$* \overset{1}{1} \times \overset{2}{2} \times \overset{1}{3} \times \overset{3}{4} \times \overset{1}{5} \times \overset{1}{6} \times \overset{3}{7} \times \overset{1}{8} \times \overset{1}{9} \times \overset{1}{10}$$

m-1

$2 \rightarrow \underline{\underline{8}} \quad 5 \rightarrow \underline{\underline{2}}$

N.O.Z = 2

m-2

$$\begin{array}{r} 2 \overline{) 10} \\ 2 \overline{) 5} \\ 2 \overline{) 2} \\ \hline 1 \end{array}$$

2 → 8 बार

$$\begin{array}{r} 5 \overline{) 10} \\ \hline 2 \end{array}$$

5 → 2 बार

N.O.Z = 2

$$* 1 \times 2 \times 3 \times 4 \times \dots \times 50$$

$$\underline{50!}^n \quad 50!$$

$$\begin{array}{r} 5 \overline{) 50} \\ 5 \overline{) 10} \\ 2 \end{array} \}^{12}$$

$$N.O.Z = \underline{\underline{12}}$$

$$* A = 60!$$

$$\underline{50!}^n$$

$$\begin{array}{r} 5 \overline{) 60} \\ 5 \overline{) 12} \\ 2 \end{array} \}^{14}$$

$$N.O.Z = \underline{\underline{14}}$$

* 100!

$$\underline{50!}^n \quad \begin{array}{l} 5 \overline{) 100} \\ 5 \overline{) 20} \\ 4 \end{array} \} 24$$

$$N.O.Z = \underline{24}$$

$$\begin{aligned} \text{ans} &= 2S \times 1 - 1 \\ &= 24 \end{aligned}$$

* 200!

$$\begin{array}{l} 5 \overline{) 200} \\ 5 \overline{) 40} \\ 5 \overline{) 8} \\ 1 \end{array} \} 49$$

$$N.O.Z = 49$$

$$\begin{aligned} \text{ans} &= 2S \times 2 - 1 \\ &= 49 \end{aligned}$$

* 300!

$$\begin{array}{l} 5 \overline{) 300} \\ 5 \overline{) 60} \\ 5 \overline{) 12} \\ 2 \end{array} \} 74$$

$$N.O.Z = 74$$

$$\begin{aligned} \text{ans} &= 2S \times 3 - 1 \\ &= 74 \end{aligned}$$

* 400!

$$\begin{array}{l} 5 \overline{) 400} \\ 5 \overline{) 80} \\ 5 \overline{) 16} \\ 3 \end{array} \} 99$$

$$N.O.Z = 99$$

$$\begin{aligned} \text{ans} &= 2S \times 4 - 1 \\ &= 99 \end{aligned}$$

2Sn-1

* 700!

$25n-1$

$$\begin{aligned}\text{ans} &= 25 \times 7 - 1 \\ &= 175 - 1 \\ &= 174\end{aligned}$$

* 900!

$25n-1$

$$\begin{aligned}\text{ans} &= 25 \times 9 - \\ &= 225 - 1 \\ &= 224 \checkmark\end{aligned}$$

$$1 \times 2 \times 3 \times \dots \times 1000 \quad 4.$$
$$= \underline{\underline{1000!}} \quad (2Sn-1)$$

$$\begin{aligned} \text{Ans} &= 2S \times 10 - 1 \\ &= 2S0 - 1 \\ &= \underline{\underline{249}} \end{aligned}$$

The number 1, 2, 3, 4, 1000 are multiplied together. The number of zeros at the end (on the right) of the product must be:

संख्याओं 1, 2, 3, 4 1000 को एक साथ गुना किया जाता है। गुणनफल के अंत (दाहिनी ओर) में शून्यों की संख्या है:

- (a) 30
(c) 211

- (b) 200
(d) ☒ 249

Direction: Find the number of zero's at the end of followings numbers or expression.

निर्देश: निम्नलिखित संख्याओं या व्यंजकों के अंत में शून्यों की संख्या ज्ञात कीजिए।

5. $A = \underline{300!}$

✓ (a) 74

(c) 75

(b) 72

(d) 76

$$2S_{n-1}$$

$$\begin{aligned} Qn\&= 2S \times 3 - 1 \\ &= 74 \end{aligned}$$

$$\begin{array}{r} 5 \overline{) 625} \\ 5 \overline{) 125} \\ 5 \overline{) 25} \\ 5 \overline{) 5} \\ 1 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 625} \\ 5 \overline{) 125} \\ 5 \overline{) 25} \\ 5 \overline{) 5} \end{array}} \right\} \underline{\underline{156}}$$

6. **B = 625!**

(a) 125

(c) 155

(b) 150

(d) ~~156~~

7. $C = 2^{\underline{502}} \times 5^{205}$

(a) 502

(c) 707

✓ (b) 205

(d) None of these

Ans = 205

8. $D = 1 \times 3 \times 5 \times 7 \times \dots \times 999$

(a) 246

(b) 199

(c) 99

(d) 0



Q. $1 \times 3 \times 5 \times 7 \times \dots \times 99 \times 2^2$

$$N \cdot O \cdot Z = \underline{\underline{2}}$$

Q. $1 \times 3 \times 5 \times \dots \times 99 \times 2^6$

$$N \cdot O \cdot Z = \underline{\underline{6}}$$

9. $E = 1 \times 3 \times 5 \times 7 \times 9 \times \dots \times 99 \times \underline{\underline{128}}$

(a) 10

(b) 12

☒ (c) 7

(d) 19

$$E = 1 \times 3 \times 5 \times \dots \times 99 \times 2^7$$

$$N.O.Z = 7$$

10. $H = 168! \times 143!$

(a) 40

(b) 34

~~(c) 74~~

(d) 1360

$$\begin{array}{r} 5 \overline{) 168} \\ 5 \overline{) 33} \\ 5 \overline{) 6} \\ 1 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 168} \\ 5 \overline{) 33} \\ 5 \overline{) 6} \\ 1 \end{array}} \right\} 40$$

$$\begin{array}{r} 5 \overline{) 143} \\ 5 \overline{) 28} \\ 5 \overline{) 5} \\ 1 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 143} \\ 5 \overline{) 28} \\ 5 \overline{) 5} \\ 1 \end{array}} \right\} 34$$

$$\begin{aligned} \text{Ans} &= 40 + 34 \\ &= 74 \end{aligned}$$

Q. $A = 132! \times 164!$

$$\begin{array}{r}
 5 \overline{) 132} \\
 5 \overline{) 26} \\
 5 \overline{) 5} \\
 1
 \end{array}
 \left. \vphantom{\begin{array}{r} 5 \overline{) 132} \\ 5 \overline{) 26} \\ 5 \overline{) 5} \\ 1 \end{array}} \right\} 32$$

$$\begin{array}{r}
 5 \overline{) 164} \\
 5 \overline{) 32} \\
 5 \overline{) 6} \\
 1
 \end{array}
 \left. \vphantom{\begin{array}{r} 5 \overline{) 164} \\ 5 \overline{) 32} \\ 5 \overline{) 6} \\ 1 \end{array}} \right\} 39$$

$A = 71$

Q $B = 398! \times 432!$

$\text{ans} = 203 //$

$$\begin{array}{r} 5 \overline{) 500} \\ 5 \overline{) 100} \\ 5 \overline{) 20} \\ 4 \end{array} \left. \vphantom{\begin{array}{r} 5 \overline{) 500} \\ 5 \overline{) 100} \\ 5 \overline{) 20} \right\} 124$$

or

$$2S_{n-1}$$

$$= 2S \times S - 1$$

$$= \boxed{124}$$

11. $I = 2 \times 4 \times 6 \times 8 \times \dots \times 1000$

- (a) 249
(b) 125
(c) 124
(d) 997

$I = \underbrace{2 \times 1} \times \underbrace{2 \times 2} \times \underbrace{2 \times 3} \times \underbrace{2 \times 4} \times \dots \times \underbrace{2 \times 500}$

$= (2 \times 2 \times 2 \times \dots) \times (1 \times 2 \times 3 \times 4 \times \dots \times 500)$

$I = 2^{500} \times \underline{\underline{500!}}$

12. $J = 5 \times 10 \times 15 \times 20 \times \dots \times 1000$

(a) 49

(b) 111

(c) 197

(d) 247

$$\begin{aligned} J &= \underline{5 \times 1} \times \underline{5 \times 2} \times \underline{5 \times 3} \times \underline{5 \times 4} \times \dots \times \underline{5 \times 200} \\ &= (5 \times 5 \times 5 \times \dots) \times (1 \times 2 \times 3 \times \dots \times 200) \\ &= \boxed{5^{200} \times 200!} \end{aligned}$$

Handwritten calculation for the exponent of 2 in 200!:

$$\begin{array}{r} 2 \overline{) 200} \\ 2 \overline{) 100} \\ 2 \overline{) 50} \\ 2 \overline{) 25} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ 2 \overline{) 3} \\ \hline \end{array}$$

Sum of quotients: 100 + 50 + 25 + 12 + 6 + 3 = 197

Handwritten calculation for the exponent of 5 in 200!:

$$\begin{array}{r} 5 \rightarrow 5 \overline{) 200} \\ 5 \overline{) 40} \\ 5 \overline{) 8} \\ \hline \end{array}$$

Sum of quotients: 40 + 8 = 48. (Note: The handwritten result is 49, which is correct as 200/5 = 40, 40/5 = 8, 8/5 = 1, 1/5 = 0, so 40+8+1 = 49)

Handwritten calculation for the final result:

$$5 \rightarrow 200 + 49 = \boxed{249}$$

$$A = 10 \times 100 \times 1000$$

Solⁿ $A = 1000000$

$$N.O.Z = \underline{\underline{6}}$$

$$B = \underline{10} + 100 + 1000$$

Solⁿ

$$\begin{array}{r} 1000 \\ + 100 \\ + 10 \\ \hline \underline{\underline{1110}} \end{array}$$

$$N.O.Z = 1$$

$$* C = \underline{\underline{100}} + 1000 + 10^9$$

$$N.O.Z$$

Ans

$$ans = \underline{\underline{2}}$$

13. $K = 10^{100} + 10^{90} + 10^{80} + \dots + 10^{20} + 10^{10}$

- ☒ (a) 10
(c) 550

- (b) 100
(d) 55

ans = minimum power
= 10

14. $L = \underline{(5!)}^{6!}$

(a) 120

(c) 600

 (b) 720

(d) 840

Ans = $(120)^{720}$
= $(12 \times 10)^{720}$
= $12^{720} \times 10^{720}$

$1! = 1$
 $2! = 2$
 $3! = 6$
 $4! = 24$
 $5! = 120$
 $6! = 720$

15. $P = 1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times \dots \times 100^{100}$

(a) 1050

(b) 1200

~~(c) 1300~~

(d) 1450

$$P = 1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times \dots \times 100^{100}$$

$$= \dots \times 5^5 \times \dots \times 10^{10} \times \dots \times 15^{15} \times \dots \times 25^{25} \times \dots \times 100^{100}$$

$$(5 + 10 + 15 + 20 + \underline{25} + \dots + 100)$$

$$n = \frac{a_n - a}{d} + 1 = \frac{100 - 5}{5} + 1 = \underline{\underline{20}}$$

$$S_n = \frac{n}{2} (a + a_n)$$

$$= \frac{20}{2} (5 + 100)$$

$$= 1050$$

$$\begin{array}{r} 25 \\ 25 \\ 25 \\ 25 \\ 25 \\ \hline 250 \end{array}$$

$$\text{ans} = 1050 + 250 = 1300$$

$$Q \ A = 1^1 \times 2^2 \times 3^3 \times 4^4 \times \dots \times 50^{50}$$

ans $5^5 \times 10^{10} \times 15^{15} \times \dots$

$$(5 + 10 + 15 + \dots + 50)$$

$$n = \frac{50 - 5}{5} + 1 = 10$$

$$S_n = \frac{n}{2} (a + a_n)$$

$$= \frac{10}{2} (5 + 50)$$

$$= 5 \times 55$$

$$= 275$$

$$\frac{25}{50}$$

$$+ \rightarrow \text{ans} = \underline{\underline{350}}$$

16. How many zeros are there in the product :

गुणनफल में कितने शून्य हैं :

$$1^{50} \times 2^{49} \times 3^{48} \times \dots \times 50^1$$

[CDS - 2021 (II)]

(a) 262

(b) 261

(c) 246

(d) 235

$$1^{50} \times 2^{49} \times 3^{48} \times 4^{47} \times 5^{46} \times \dots \times 10^{41} \times \dots \times 15^{36} \times \dots \times 20^{31} \times \dots \times 25^{26} \times \dots \times 50^1$$

$$5^{46} \times 10^{41} \times 15^{36} \times \dots \times 50^1$$

$$(46 + 41 + 36 + 31 + 26 + \dots + 1)$$

$$n = \frac{1-46}{-5} + 1 = 10$$

$$S_n = \frac{10}{2} (46 + 1) = 5 \times 47 = 235$$

$$\begin{array}{r} 26 \\ + 26 \\ \hline 52 \end{array}$$

$$\begin{array}{r} 235 \\ 26 \\ \hline 262 \end{array}$$

17. $Q = 100^1 \times 99^2 \times 98^3 \times \dots \times 2^{99} \times 1^{100}$

- (a) 1124 (b) 1120
(c) 970 (d) 1121

$100^1 \times 99^2 \times \dots \times 1^{100}$
 $5^{96} 10^{91} 15^{86} \dots 100^1$
 $\checkmark (96 + 91 + 86 + \dots + 1)$
 $n = 20$
 $S_n = \frac{n}{2}(96 + 1)$
 $= 970$

$25^{76} 50^{51} 75^{26} 100^1$
$$\begin{array}{r} 76 \\ + 51 \\ + 26 \\ + 1 \\ \hline 154 \end{array}$$

$$\begin{array}{r} \text{ans} = 970 + 154 \\ = 1124 \end{array}$$

18. What is the highest power of 4 in 100! ?

100! में 4 की उच्चतम घात क्या है?

(a) 32

(c) 49

☒ (b) 48

(d) None of these

$$\begin{array}{r} 2 \overline{) 100} \\ 2 \overline{) 50} \\ 2 \overline{) 25} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ 2 \overline{) 3} \\ 1 \end{array} \quad \left. \vphantom{\begin{array}{r} 2 \overline{) 100} \\ 2 \overline{) 50} \\ 2 \overline{) 25} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ 2 \overline{) 3} \\ 1 \end{array}} \right\} 97$$

$$100! \rightsquigarrow 2^{97} \approx 2^{96}$$

$$\text{ans} = \frac{96}{2} = 48$$

$$\begin{aligned} 2^2 &= 4 \\ 2^{96} &= 4 \end{aligned}$$

$$100! \rightarrow N \cdot O \cdot Z \rightarrow 10? = 2 \times 5$$

$$\begin{array}{r} 2 \overline{) 100} \\ 2 \overline{) 50} \\ 2 \overline{) 25} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ 2 \overline{) 3} \\ 1 \end{array} \quad \left. \vphantom{\begin{array}{r} 2 \overline{) 100} \\ 2 \overline{) 50} \\ 2 \overline{) 25} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ 2 \overline{) 3} \\ 1 \end{array}} \right\} 97$$

$$\text{ans} = \underline{\underline{24}}$$

$$\begin{array}{r} 5 \overline{) 100} \\ 5 \overline{) 20} \\ 4 \end{array} \quad \left. \vphantom{\begin{array}{r} 5 \overline{) 100} \\ 5 \overline{) 20} \\ 4 \end{array}} \right\} \underline{\underline{24}}$$

$$100! \rightsquigarrow 6 = ? \rightarrow 2 \times 3 \underline{\underline{=}}$$

$$\begin{array}{r} 2 \overline{) 100} \\ 2 \overline{) 50} \\ 2 \overline{) 25} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ 2 \overline{) 3} \\ 1 \end{array} \quad \left. \vphantom{\begin{array}{r} 2 \overline{) 100} \\ 2 \overline{) 50} \\ 2 \overline{) 25} \\ 2 \overline{) 12} \\ 2 \overline{) 6} \\ 2 \overline{) 3} \\ 1 \end{array}} \right\} 97$$

$$\text{ans} = \underline{\underline{48}}$$

$$\begin{array}{r} 3 \overline{) 100} \\ 3 \overline{) 33} \\ 3 \overline{) 11} \\ 3 \overline{) 3} \\ 1 \end{array} \quad \left. \vphantom{\begin{array}{r} 3 \overline{) 100} \\ 3 \overline{) 33} \\ 3 \overline{) 11} \\ 3 \overline{) 3} \\ 1 \end{array}} \right\} 48$$

What is the highest power of 72 in 72! ?

72! में 72 की उच्चतम घात क्या है?

- (a) 20
- (b) 21
- (c) 19
- (d) 24

$$72 = 2^3 \times 3^2$$

$$\begin{array}{r} 2 \overline{) 72} \\ 2 \overline{) 36} \\ 2 \overline{) 18} \\ 2 \overline{) 9} \\ 2 \overline{) 4} \\ 2 \overline{) 2} \\ \hline \end{array} \quad \left. \begin{array}{l} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{array} \right\} 10$$

$$= 2^{10} \quad \left| \quad = 3^{17} \right.$$
$$= (2^3)^{23} \cdot 2^1 \quad \left| \quad = (3^2)^{17} \right.$$

Ans = 17

$$\begin{array}{r} 3 \overline{) 72} \\ 3 \overline{) 24} \\ 3 \overline{) 8} \\ \hline 2 \end{array} \quad \left. \begin{array}{l} 3 \\ 3 \\ 3 \end{array} \right\} 34$$

19. What is the highest power of 72 in 100! ?

100! में 72 की उच्चतम घात क्या है?

(a) 20

(b) 21

(c) 19

(d) 24

$$72 = 2^3 \times \underline{\underline{3^2}}$$

$$3^{48} = (3^2)^{24} \underline{\underline{=}}$$

$$\begin{array}{r} 3 \overline{) 100} \\ 3 \overline{) 33} \\ 3 \overline{) 11} \\ 3 \overline{) 3} \\ \underline{1} \end{array} \left. \vphantom{\begin{array}{r} 3 \overline{) 100} \\ 3 \overline{) 33} \\ 3 \overline{) 11} \\ 3 \overline{) 3} \\ \underline{1} \end{array}} \right\} 48$$