ALGBERA

बीजगणित

SHEET-01

COMPLETE CLASS NOTES BY ADITYA RANJAN

Type: 1

$$x + \frac{1}{x} \quad \& \quad x - \frac{1}{x}$$

$$\boxed{x+\frac{1}{x}=n}$$

$$* \; x^2+\frac{1}{x^2} = n^2-2$$

$$* \; x^3+\frac{1}{x^3} = n^3-3n$$

$$* \; x^4+\frac{1}{x^4} = (n^2-2)^2-2$$

$$* \; x^5+\frac{1}{x^5} =$$

$$* \; x^6+\frac{1}{x^6} = (n^3-3n)^2-2$$

$x + \frac{1}{x} = n$

<u>Proof</u>: $\left(x + \frac{1}{x}\right)^2 = n^2$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 \times \cancel{x} \times \frac{1}{\cancel{x}} = n^2$

$\Rightarrow \boxed{x^2 + \frac{1}{x^2} = n^2 - 2}$

Q. If $x+\frac{1}{x}=3$ $\quad x^2+\frac{1}{x^2}=?$

Ans $\quad x^2+\frac{1}{x^2}=n^2-2$

$$=3^2-2=7$$

Q. If $x+\frac{1}{x}=7$ $\quad x^2+\frac{1}{x^2}=?$

Ans $\quad x^2+\frac{1}{x^2}=n^2-2$

$$=(7)^2-2$$

$$=\underline{\underline{47}}$$

$x+\frac{1}{x}=n$

<u>Proof:</u>

$$\left(x+\frac{1}{x}\right)^3=n^3$$

$$(a+b)^3=a^3+b^3+3ab(a+b)$$

$$\Rightarrow x^3+\frac{1}{x^3}+3\times\cancel{x}\times\frac{1}{\cancel{x}}\left(x+\frac{1}{x}\right)=n^3$$

$$\Rightarrow x^3+\frac{1}{x^3}+3n=n^3$$

$$\boxed{\therefore x^3+\frac{1}{x^3}=n^3-3n}$$

Q. $x+\frac{1}{x}=3$ $\quad x^3+\frac{1}{x^3}=?$

Ans

$$x^3+\frac{1}{x^3}=n^3-3n$$
$$=27-9=18$$

Q. $x+\frac{1}{x}=\sqrt{5}$ $\quad x^3+\frac{1}{x^3}=?$

Ans

$$x^3+\frac{1}{x^3}=n^3-3n$$
$$=(\sqrt{5})^3-3\sqrt{5}$$
$$=5\sqrt{5}-3\sqrt{5}$$
$$=2\sqrt{5}$$

$$(\sqrt{5})^3=\underbrace{\sqrt{5}\times\sqrt{5}}\times\sqrt{5}$$
$$=5\sqrt{5}$$

Case of Double Power

* $x^1 + \frac{1}{x^1} = n$
* $x^2 + \frac{1}{x^2} = n^2 - 2$
* $x^4 + \frac{1}{x^4} =$
* $x^8 + \frac{1}{x^8} =$
* $x^{16} + \frac{1}{x^{16}} =$

Ex:- * $x^1 + \frac{1}{x^1} = 3$

* $x^2 + \frac{1}{x^2} = (3)^2 - 2 = 7$

* $x^4 + \frac{1}{x^4} = (7)^2 - 2 = 47$

* $x^8 + \frac{1}{x^8} = (47)^2 - 2$

$= 2209 - 2$

$= \underline{\underline{2207}}$

* $x + \frac{1}{x} = 1$

$x^2 + \frac{1}{x^2} = (1)^2 - 2 = 1 - 2 = -1$

$x^4 + \frac{1}{x^4} = (-1)^2 - 2 = 1 - 2 = -1$

$x^8 + \frac{1}{x^8} = (-1)^2 - 2 = 1 - 2 = -1$

$* \; x^3 + \frac{1}{x^3} = 18$

$* \; x^6 + \frac{1}{x^6} = (18)^2 - 2 = 324 - 2 = 322$

$* \; x^{12} + \frac{1}{x^{12}} = (322)^2 - 2$

$= 103684 - 2$

$= \underline{103682}$

$\vdots$

$$\begin{array}{r} 322 \\ \times\ 322 \\ \hline 103684 \end{array}$$

$$x+\frac{1}{x}$$

Even power $\rightarrow$ $2, 4, 6, 8 - - - -$

$$\left(x^2+\frac{1}{x^2}\right)\left(x^4+\frac{1}{x^4}\right)\left(x^6+\frac{1}{x^6}\right)\left(x^8+\frac{1}{x^8}\right) - -$$

(n^2-2)

Odd powers $\rightarrow$ $3, 5, 7, 9 - - -$

$$\left(x^3+\frac{1}{x^3}\right) ** \left(x^5+\frac{1}{x^5}\right) ** \left(x^7+\frac{1}{x^7}\right)$$

(n^3-3n)

odd powers

$$* \; x^5+\frac{1}{x^5} = \left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$$

$$= x^5+\frac{x^2}{x^3}+\frac{x^3}{x^2}+\frac{1}{x^5}-x-\frac{1}{x}$$

$$= x^5+\cancel{\frac{1}{x}}+\cancel{x}+\frac{1}{x^5}-\cancel{x}-\cancel{\frac{1}{x}}$$

$$= x^5+\frac{1}{x^5}$$

$$x^5+\frac{1}{x^5} = (n^2-2)(n^3-3n)-n$$

* $x+\frac{1}{x}=n$

* $x^2+\frac{1}{x^2}=n^2-2$

* $x^3+\frac{1}{x^3}=n^3-3n$

* $x^4+\frac{1}{x^4}=(n^2-2)^2-2$

* $x^5+\frac{1}{x^5}=(n^2-2)(n^3-3n)-n$

* $x^6+\frac{1}{x^6}=(n^3-3n)^2-2$

* $x^7+\frac{1}{x^7}=\left(x^3+\frac{1}{x^3}\right)\left(x^4+\frac{1}{x^4}\right)-\left(x+\frac{1}{x}\right)$

* $x + \frac{1}{x} = 4$

* $x^2 + \frac{1}{x^2} = (4)^2 - 2 = 14$

* $x^3 + \frac{1}{x^3} = (4)^3 - 3 \times 4$
$= 64 - 12 = 52$

* $x^4 + \frac{1}{x^4} = (14)^2 - 2$
$= 196 - 2$
$= 194$

* $x^5 + \frac{1}{x^5} = 14 \times 52 - 4$
$= 728 - 4 = 724$

* $x^6 + \frac{1}{x^6} = (52)^2 - 2$
$= 2704 - 2 = 2702$

* $x^7 + \frac{1}{x^7} = 52 \times 194 - 4$
$= 10088 - 4$
$= \underline{10084}$

1. If $x^1 + \frac{1}{x} = 3$ Find

(i) $x^2 + \frac{1}{x^2}$

(ii) $x^3 + \frac{1}{x^3}$

(iii) $x^4 + \frac{1}{x^4}$

(iv) $x^5 + \frac{1}{x^5}$

(v) $x^6 + \frac{1}{x^6}$

(vi) $x^7 + \frac{1}{x^7}$

$x^2 + \frac{1}{x^2} = (3)^2 - 2 = 7$

$x^3 + \frac{1}{x^3} = (3)^3 - 3 \times 3 = 18$

$x^4 + \frac{1}{x^4} = (7)^2 - 2 = 47$

$x^5 + \frac{1}{x^5} = 7 \times 18 - 3 = 123$

$x^6 + \frac{1}{x^6} = (18)^2 - 2 = 322$

$x^7 + \frac{1}{x^7} = 18 \times 47 - 3 = 846 - 3 = 843$

If $x+\frac{1}{x}=n$

* $x^2+\frac{1}{x^2} = n^2-2+2$
* $x^3+\frac{1}{x^3} = n^3-3n$
* $x^4+\frac{1}{x^4} = (n^2-2)^2-2+2$
* $x^5+\frac{1}{x^5} = (n^2-2)(n^3-3n)-n$
* $x^6+\frac{1}{x^6} = (n^3-3n)^2-2$

If $x-\frac{1}{x}=n$

* $x^2+\frac{1}{x^2} = n^2+2$
* $x^3-\frac{1}{x^3} = n^3+3n$
* $x^4+\frac{1}{x^4} = (n^2+2)^2-2$
* $x^5-\frac{1}{x^5} = (n^2+2)(n^3+3n)-n$

*** $x^6+\frac{1}{x^6} = (n^2+3n)^2+2$

* $x - \frac{1}{x} = n$

$$\left(x - \frac{1}{x}\right)^2 = (n)^2$$

$$\Rightarrow x^2 + \left(\frac{1}{x}\right)^2 - 2 \times \cancel{x} \times \frac{1}{\cancel{x}} = n^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = n^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = n^2 + 2$$

$$* \quad x^3 - \frac{1}{x^3} = (n^3 + 3n)$$

$$\Rightarrow \left[x^3 - \frac{1}{x^3}\right]^2 = (n^3 + 3n)^2$$

$$\Rightarrow x^6 + \frac{1}{x^6} - 2 \times \cancel{x^3} \times \frac{1}{\cancel{x^3}} = (n^3 + 3n)^2$$

$$\Rightarrow x^6 + \frac{1}{x^6} = (n^3 + 3n)^2 + 2$$

* $x - \frac{1}{x} = 2$

* $x^2 + \frac{1}{x^2} = n^2 + 2 = 6$

* $x^3 - \frac{1}{x^3} = n^3 + 3n = 14$

* $x^4 + \frac{1}{x^4} = (6)^2 - 2 = 34$

* $x^5 - \frac{1}{x^5} = 6 \times 14 - 2 = 82$

* $x^6 + \frac{1}{x^6} = (14)^2 + 2 = \underline{198}$

$x - \frac{1}{x} = 4$

2. If $x - \frac{1}{x} = 4$, Find

(i) $x^2 + \frac{1}{x^2}$

~~(ii)~~ $x^2 - \frac{1}{x^2}$

(iii) $x^3 - \frac{1}{x^3}$

(iv) $x^4 + \frac{1}{x^4}$

(v) $x^5 - \frac{1}{x^5}$

(vi) $x^6 + \frac{1}{x^6}$

* $x^2 + \frac{1}{x^2} = 4^2 + 2 = 18$

* $x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4 = 76$

* $x^4 + \frac{1}{x^4} = 18^2 - 2 = 322$

* $x^5 - \frac{1}{x^5} = 18 \times 76 - 4$
$= 1368 - 4 = 1364$

* $x^6 + \frac{1}{x^6} = (76)^2 + 2$
$= 5776 + 2$
$= 5778$

$a + \frac{1}{a} = 3$

$a^2 + \frac{1}{a^2} = 3^2 - 2 = 7$

$a^4 + \frac{1}{a^4} = 7^2 - 2$

$= 47$

3. If $a + \frac{1}{a} = 3$, then the value of $a^4 + \frac{1}{a^4}$ is:

यदि $a + \frac{1}{a} = 3$ है, तो $a^4 + \frac{1}{a^4}$ का मान क्या होगा?

SSC CPO 04/10/2023 (Shift-01)

(a) 27
(b) 81
(c) 48
(d) 47

$a^4 + \frac{1}{a^4} = (n^2-2)^2 - 2$

$= (7)^2 - 2$

$= 47$

3. If $a + \frac{1}{a} = 3$, then the value of $a^4 + \frac{1}{a^4}$ is:

यदि $a + \frac{1}{a} = 3$ है, तो $a^4 + \frac{1}{a^4}$ का मान क्या होगा?

SSC CPO 04/10/2023 (Shift-01)

(a) 27
(b) 81
(c) 48
(d) 47

3. If $a + \frac{1}{a} = 3$, then the value of $a^4 + \frac{1}{a^4}$ is:

यदि $a + \frac{1}{a} = 3$ है, तो $a^4 + \frac{1}{a^4}$ का मान क्या होगा?

SSC CPO 04/10/2023 (Shift-01)

(a) 27 +2 = 29 ✗

(b) 81 +2 = 83 ✗

(c) 48 +2 = 50 ✗

(d) 47 +2 = 49 ✓

4. If $a + \frac{1}{a} = 7$, then $a^5 + \frac{1}{a^5}$ is equal to:

यदि $a + \frac{1}{a} = 7$ है, तो $a^5 + \frac{1}{a^5}$ निम्न में से किसके बराबर है?

SSC CGL 17/07/2023 (Shift-02)

(a) 15127 (b) 13127

(c) 14527 (d) 11512

$a^5 + \frac{1}{a^5} = (n^2-2)(n^3-3n) - n$

$= 47 \times 322 - 7$

$= 15134 - 7$

$= 15127$

5. If $x + \frac{1}{x} = -6$, what will be the value of $x^5 + \frac{1}{x^5}$

यदि $x + \frac{1}{x} = -6$, है तो $x^5 + \frac{1}{x^5}$ का मान क्या होगा?

SSC CGL 17/07/2023 (Shift-03)

(a) –7776 (b) –6726

(c) –6730 (d) –6732

$x^5 + \frac{1}{x^5} = (n^2-2)(n^3-3n) - n$

$= (36-2)(-216+18) - (-6)$

$= 34(-198) + 6$

$= -6732 + 6$

$= -6726$

$a+\frac{1}{a}=3$

6. If $a+\frac{1}{a}=3$, then $a^6+\frac{1}{a^6}$ is equal to :

$a^6+\frac{1}{a^6}=(n^3-3n)^2-2$

$=(27-9)^2-2$

$=324-2$

$=322$

SSC CHSL 10/07/2019 (Shift- 02)

(a) 319 (b) 322

(c) 780 (d) 730

M-1

$x - \frac{1}{x} = 10$

$x^4 + \frac{1}{x^4} = (n^2+2)^2 - 2$

$= (102)^2 - 2$

$= 10404 - 2$

$= 10402$

7. If $\left(x - \frac{1}{x}\right) = 10$, what is the value of $x^4 + \frac{1}{x^4}$?

यदि $\left(x - \frac{1}{x}\right) = 10$ है, तो $x^4 + \frac{1}{x^4}$ का मान क्या है?

SSC CGL 26/07/2023 (Shift-03)

(a) 10404 (b) 10402

(c) 10406 (d) 10400

8. If $x - \frac{1}{x} = 10$, then $x^3 - \frac{1}{x^3}$ is equal to :

SSC CHSL 4/07/2019 (Shift-02)

(a) 970 (b) 1000

(c) 1030 (d) 1100

$x - \frac{1}{x} = n$

$x^3 - \frac{1}{x^3} = n^3 + 3n$

ans $= n^3 + 3n$

$= (10)^3 + 3 \times 10$

$= 1000 + 30$

$= \underline{1030}$

$3\left(y-\frac{1}{y}\right)=5$

$\Rightarrow y-\frac{1}{y}=\frac{5}{3}$

$\therefore y^2+\frac{1}{y^2}=n^2+2$

$=\left(\frac{5}{3}\right)^2+2$

$=\frac{25}{9}+2$

$=\frac{43}{9}$

9. If $\left(3y-\frac{3}{y}\right)=5$, find the value of $\left(y^2+\frac{1}{y^2}\right)$.

यदि $\left(3y-\frac{3}{y}\right)=5$ है, तो $\left(y^2+\frac{1}{y^2}\right)$ का मान ज्ञात करें।

SSC CGL 19/07/2023 (Shift-02)

(a) $\frac{47}{9}$

(b) $\frac{49}{9}$

(c) $\frac{41}{9}$

(d) $\frac{43}{9}$ ✓

10. If $x - \frac{1}{x} = 13$, what will be the value of $x^4 + \frac{1}{x^4}$?

यदि $x - \frac{1}{x} = 13$ है, तो $x^4 + \frac{1}{x^4}$ का मान क्या होगा?

SSC CGL 03/12/2022 (Shift-01)

(a) 28561 (b) 29243

(c) 27887 (d) 29239

$x - \frac{1}{x} = n$

$x^4 + \frac{1}{x^4} = (n^2+2)^2 - 2$

ans $= (n^2+2)^2 - 2$

$= (13^2+2)^2 - 2$

$= (171)^2 - 2$

$= 29241 - 2$

$= 29239$

ans = 24 × 140 − 5
= 3480 − 5
= 3775

11. If $\left(x-\frac{1}{x}\right)=5$, then the value of $\left(x^5-\frac{1}{x^5}\right)$ is :

यदि $\left(x-\frac{1}{x}\right)=5$, तो $\left(x^5-\frac{1}{x^5}\right)$ का मान है:

SSC CHSL 07/08/2023 Shift-02

(a) ~~2535~~
(b) ~~2540~~
(c) ~~2515~~
(d) ~~2525~~

$x-\frac{1}{x}=n$

$x^5-\frac{1}{x^5}=(n^2+2)(n^3+3n)-n$

12. If $x - \frac{1}{x} = -6$, what will be the value of $x^5 - \frac{1}{x^5}$?

यदि $x - \frac{1}{x} = -6$ है, तो $x^5 - \frac{1}{x^5}$ का मान क्या होगा?

SSC CGL 18/07/2023 (Shift-02)

(a) – 8898 (b) – 8896

(c) – 8886 (d) – 8892

ans $= (36+2)(-216-18)-(-6)$

$= 38(-234)+6$

$= -8892+6$

$= -8886$

$x - \frac{1}{x} = n$

$x^5 - \frac{1}{x^5} = (n^2+2)(n^3+3n)-n$

13. If $\left(y-\frac{1}{y}\right)=4$, find the value of $\left(y^6+\frac{1}{y^6}\right)$.

यदि $\left(y-\frac{1}{y}\right)=4$, है तो $\left(y^6+\frac{1}{y^6}\right)$ का मान ज्ञात करें

SSC CGL 25/07/2023 (Shift-01)

(a) 5774 (b) 4096

(c) 5776 (d) 5778

$y-\frac{1}{y}=n$

$y^6+\frac{1}{y^6}=(n^3+3n)^2+2$

ans $=(64+12)^2+2$

$=(76)^2+2$

$=5776+2$

$=5778$

Questions related to root

$$(\sqrt{x})^2 = \sqrt{x} \times \sqrt{x} = x$$

$$(\sqrt{x})^3 = \underbrace{\sqrt{x} \times \sqrt{x}} \times \sqrt{x}$$

$$= x\sqrt{x}$$

$$(\sqrt{5})^2 = 5$$

$$(\sqrt{5})^3 = 5\sqrt{5}$$

14. If $\sqrt{x}-\frac{1}{\sqrt{x}}=4$, then $x^2+\frac{1}{x^2}$ is equal to :

SSC CGL 10/06/2019 (Shift-01)

(a) 192 (b) 326
(c) 322 (d) 256

$$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = 4^2$$

$$\Rightarrow (\sqrt{x})^2+\left(\frac{1}{\sqrt{x}}\right)^2-2\times\cancel{\sqrt{x}}\times\frac{1}{\cancel{\sqrt{x}}} = 16$$

$$\Rightarrow x+\frac{1}{x}-2=16$$

$$\Rightarrow x+\frac{1}{x}=18$$

$$x^2+\frac{1}{x^2}=(18)^2-2$$
$$=324-2$$
$$=322$$

$$(a-b)^2=a^2+b^2-2ab$$

$$x+\frac{1}{x}=n$$
$$x^2+\frac{1}{x^2}=n^2-2$$

$$x-\frac{1}{x}=n$$
$$x^2+\frac{1}{x^2}=n^2+2$$

$\sqrt{x} - \frac{1}{\sqrt{x}} = 4$

14. If $\sqrt{x} - \frac{1}{\sqrt{x}} = 4$, then $x^2 + \frac{1}{x^2}$ is equal to :

SSC CGL 10/06/2019 (Shift-01)

(a) 192 (b) 326
(c) 322 (d) 256

$x + \frac{1}{x} = (4)^2 + 2 = 18$

$x^2 + \frac{1}{x^2} = (18)^2 - 2$
$= 322$

$(a-b)^2 = a^2 + b^2 - 2ab$

$x + \frac{1}{x} = n$
$x^2 + \frac{1}{x^2} = n^2 - 2$

$x - \frac{1}{x} = n$
$x^2 + \frac{1}{x^2} = n^2 + 2$

14. If $\sqrt{x}-\frac{1}{\sqrt{x}}=4$, then $x^2+\frac{1}{x^2}$ is equal to :

SSC CGL 10/06/2019 (Shift-01)

(a) 192 +2
(b) 326 +2
(c) 322 +2
(d) 256 +2

15. If $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{6}$, then $x^2 + \frac{1}{x^2}$ is equal to :

SSC CGL 11/06/2019 (Shift-02)

(a) 62 +2=64
(b) 40
(c) 54
(d) 66

15. If $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{6}$, then $x^2 + \frac{1}{x^2}$ is equal to :

SSC CGL 11/06/2019 (Shift-02)

(a) 62 (b) 40
(c) 54 (d) 66

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = (\sqrt{6})^2$$

$$\Rightarrow x + \frac{1}{x} - 2 \times \cancel{\sqrt{x}} \times \frac{1}{\cancel{\sqrt{x}}} = 6$$

$$\Rightarrow x + \frac{1}{x} = 8$$

$$x^2 + \frac{1}{x^2} = n^2 - 2$$

$$= 8^2 - 2$$

$$= 62$$

Q. $x + \frac{1}{x} = 3\sqrt{3}$

$x^2 + \frac{1}{x^2} = ?$

(a) 24

(b) 23

(c) 25

(d) 27

Ans

$= n^2 - 2$

$= (3\sqrt{3})^2 - 2$

$= 9 \times 3 - 2$

$= 25$

Concept of Bridging

$(a+b)^2 = (a-b)^2 + 4ab$

$(a-b)^2 = (a+b)^2 - 4ab$

If $x - \frac{1}{x} = n$ then $x + \frac{1}{x} = \sqrt{n^2+4}$

If $x + \frac{1}{x} = n$ then $x - \frac{1}{x} = \sqrt{n^2-4}$

Q. If $x+\frac{1}{x}=3$ then $x-\frac{1}{x}=\sqrt{n^2-4}$
$=\sqrt{9-4}=\sqrt{5}$

Q. If $x+\frac{1}{x}=7$ then $x-\frac{1}{x}=\sqrt{n^2-4}$
$=\sqrt{49-4}=\sqrt{45}=\sqrt{9\times5}=3\sqrt{5}$

Q. If $x-\frac{1}{x}=4$ then $x+\frac{1}{x}=\sqrt{n^2+4}$ $=\sqrt{16+4}=\sqrt{20}=2\sqrt{5}$

Q. If $x-\frac{1}{x}=3$ then $x+\frac{1}{x}=\sqrt{n^2+4}$ $=\sqrt{9+4}=\sqrt{13}$

16. If $a - \frac{1}{a} = 4$, then the value of $a + \frac{1}{a}$ is:

यदि $a - \frac{1}{a} = 4$ है, तो $a + \frac{1}{a}$ का मान है:

SSC CPO 05.10.2023 (Shift-2)

(a) $5\sqrt{5}$

(b) $4\sqrt{5}$

(c) $2\sqrt{5}$

(d) $3\sqrt{5}$

$a + \frac{1}{a} = \sqrt{n^2 + 4}$

$= \sqrt{16+4} = \sqrt{20}$

$= \sqrt{4 \times 5}$

$= 2\sqrt{5}$

Q. $a+\frac{1}{a}=3$ then $a^2-\frac{1}{a^2}=?$

$$\downarrow$$

$$\left(a-\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$$

$$\boxed{a^2-b^2=(a-b)(a+b)}$$

Sol^n $a+\frac{1}{a}=3$

$$a-\frac{1}{a}=\sqrt{(3)^2-4}=\sqrt{5}$$

$$\therefore\ a^2-\frac{1}{a^2}=\left(a-\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$$

$$=\sqrt{5}\times 3$$

$$=3\sqrt{5}$$

17. If $\left(x-\frac{1}{x}\right)=0.4$, and x>0, what is the value of $\left(x^2-\frac{1}{x^2}\right)$?

यदि $\left(x-\frac{1}{x}\right)=0.4$, और $x>0$ तो $\left(x^2-\frac{1}{x^2}\right)$ का मान क्या है?

SSC CHSL 07/08/2023 Shift-02

(a) 4.16 (b) 3.32

(c) 1.84 (d) 2.16

$$x^2-\frac{1}{x^2}=\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

$$=\frac{2}{5}\sqrt{\left(\frac{2}{5}\right)^2+4}$$

$$=\frac{2}{5}\sqrt{\frac{4}{25}+4}$$

$$=\frac{2}{5}\sqrt{\frac{104}{25}}$$

$$=\frac{2}{5}\times\frac{2\sqrt{26}}{5}$$

$$=\frac{4}{25}\sqrt{26}$$

$* \quad x + \frac{1}{x} = 3 \qquad x^2 + \frac{1}{x^2} = 3^2 - 2 = 7$

$* \quad x^{10} + \frac{1}{x^{10}} = 3 \qquad x^{20} + \frac{1}{x^{20}} = 3^2 - 2 = 7$

$* \quad x^3 + \frac{1}{x^3} = 5 \qquad x^6 + \frac{1}{x^6} = (5)^2 - 2 = \underline{\underline{23}}$

18. If $\left(x^2+\frac{1}{x^2}\right)=6$ and $0<x<1$, what is the value of $x^4-\frac{1}{x^4}$?

यदि $\left(x^2+\frac{1}{x^2}\right)=6$ और $0<x<1$ है, तो $x^4-\frac{1}{x^4}$ का मान क्या होगा?

SSC CGL 26/07/2023 (Shift-01)

(a) $24\sqrt{2}$ (b) $-24\sqrt{2}$

(c) $-12\sqrt{10}$ (d) $12\sqrt{10}$

let $x^2=a$

$a+\frac{1}{a}=6$, $a^2-\frac{1}{a^2}=?$

$a^2-\frac{1}{a^2}=\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$

$=6\sqrt{36-4}$

$=6\times4\sqrt{2}$

$=24\sqrt{2}$

$a - \frac{1}{a} = 4\sqrt{2}$; $a^2 - \frac{1}{a^2} = ?$

$a^2 - \frac{1}{a^2} = \left(a - \frac{1}{a}\right)\left(a + \frac{1}{a}\right)$

$= 4\sqrt{2}\sqrt{(4\sqrt{2})^2 + 4}$

$= 4\sqrt{2}\sqrt{16 \times 2 + 4}$

$= 4\sqrt{2} \times 6$

$= 24\sqrt{2}$

19. If $\overset{a}{x^2} - \frac{1}{x^2} = 4\sqrt{2}$, what is the value of $\overset{a^2}{x^4} - \frac{1}{x^4}$?

यदि $x^2 - \frac{1}{x^2} = 4\sqrt{2}$ है, तो $x^4 - \frac{1}{x^4}$ का मान क्या है?

SSC CGL 27/07/2023 (Shift-02)

(a) $16\sqrt{2}$ (b) $8\sqrt{2}$

(c) $24\sqrt{2}$ (d) $32\sqrt{2}$

Expansion

* $x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

* $x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

* $x^8 - \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

* $x^6 - \frac{1}{x^6} = (x^3)^2 - \frac{1}{(x^3)^2} = \left(x^3 + \frac{1}{x^3}\right)\left(x^3 - \frac{1}{x^3}\right)$

Expansion

$$* \quad x^4 - \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= \left(x^2 + \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$$

$$= \left((x^2)^2 - \frac{1}{(x^2)^2}\right)$$

$$= x^4 - \frac{1}{x^4}$$

* If $x+\frac{1}{x}=3$

(a) $x^2-\frac{1}{x^2} = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$

$= 3\times\left(\sqrt{3^2-4}\right)$

$= 3\times\sqrt{5}$

$= 3\sqrt{5}$

$x+\frac{1}{x}=n \quad x-\frac{1}{x}=\sqrt{n^2-4}$

* If $x+\frac{1}{x}=3$

(b) $x^4-\frac{1}{x^4} = \left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$

$= (3^2-2) \times 3 \times \left(\sqrt{3^2-4}\right)$

$= 7 \times 3 \times \sqrt{5}$

$= 21\sqrt{5}$

* If $x - \frac{1}{x} = 3$

$11^2 - 2 \qquad (\sqrt{13})^2 - 2 \qquad \sqrt{3^2+4}$

(d) $x^8 - \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

$= 119 \times 11 \times \sqrt{13} \times 3$

$= 357 \times 11 \times \sqrt{13}$

$= 3927\sqrt{13}$ ✓

* If $x - \frac{1}{x} = 2$

find $x^8 - \frac{1}{x^8} = \underbrace{\left(x^4 + \frac{1}{x^4}\right)}_{6^2-2}\underbrace{\left(x^2 + \frac{1}{x^2}\right)}_{(\sqrt{8})^2-2}\underbrace{\left(x + \frac{1}{x}\right)}_{\sqrt{n^2+4}}\underbrace{\left(x - \frac{1}{x}\right)}_{n}$

$= 34 \times 6 \times \sqrt{8} \times 2$

$= 34 \times 12 \times 2\sqrt{2}$

$= \underline{816\sqrt{2}}$

$* \quad x - \frac{1}{x} = 4$

$$x^8 - \frac{1}{x^8} = \underbrace{\left(x^4 + \frac{1}{x^4}\right)}_{322}\underbrace{\left(x^2 + \frac{1}{x^2}\right)}_{(18)^2-2}\underbrace{\left(x + \frac{1}{x}\right)}_{(\sqrt{20})^2-2}\underbrace{\left(x - \frac{1}{x}\right)}_{\sqrt{4^2+4}}$$

$$= 322 \times 18 \times \sqrt{20} \times 4$$

$$= 322 \times 72 \times 2\sqrt{5}$$

$$= 644 \times 72\sqrt{5}$$

$$= 46368\sqrt{5}$$

Q. $x+\frac{1}{x}=3$

$$x^6-\frac{1}{x^6}=\left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right)$$

$$=18\times 8\sqrt{5}$$

$$=\underline{\underline{144\sqrt{5}}}$$

$$x+\frac{1}{x}=3$$

$$x^3+\frac{1}{x^3}=(3)^3-3\times 3=18$$

$$x-\frac{1}{x}=\sqrt{3^2-4}=\sqrt{5}$$

$$x^3-\frac{1}{x^3}=(\sqrt{5})^3+3\sqrt{5}$$

$$=5\sqrt{5}+3\sqrt{5}$$

$$=8\sqrt{5}$$

Q. $x+\frac{1}{x}=3$

$$x^6-\frac{1}{x^6} = \left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right)$$

$$= 18 \times \sqrt{18^2-4}$$

$$= 18 \times \sqrt{320}$$

$$= 18 \times 8\sqrt{5}$$

$$= 144\sqrt{5}$$

$$x+\frac{1}{x}=3$$

$$x^3+\frac{1}{x^3} = n^3-3n$$

$$= 27-9$$

$$= 18$$

Q. $x+\frac{1}{x}=4$

$$x^6-\frac{1}{x^6}=\left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right)$$

$$=52\times30\sqrt{3}$$

$$=\underline{1560\sqrt{3}}$$

$$x+\frac{1}{x}=4$$

$$x^3+\frac{1}{x^3}=4^3-3\times4$$

$$=52$$

$$x-\frac{1}{x}=\sqrt{4^2-4}=\sqrt{12}$$

$$=2\sqrt{3}$$

$$x^3-\frac{1}{x^3}=(2\sqrt{3})^3+3\times2\sqrt{3}$$

$$=8\times3\sqrt{3}+6\sqrt{3}$$

$$=30\sqrt{3}$$

Q. $x+\frac{1}{x}=4$

$$x^6-\frac{1}{x^6}=\left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right)$$

$$=52\times\sqrt{(52)^2-4}$$

$$=52\times\sqrt{2704-4}$$

$$=52\times\sqrt{2700}$$

$$=52\times\sqrt{900\times3}$$

$$=52\times30\sqrt{3}$$

$$=1560\sqrt{3}$$

$$x+\frac{1}{x}=4$$

$$x^3+\frac{1}{x^3}=4^3-3\times4$$

$$=52$$

20. If $\left(x+\frac{1}{x}\right)=5\sqrt{2}$, and $x>1$, what is the value of $\left(x^6-\frac{1}{x^6}\right)$?

यदि $\left(x+\frac{1}{x}\right)=5\sqrt{2}$, और $x>1$, तो $\left(x^6-\frac{1}{x^6}\right)$ का मान क्या है?

SSC CHSL 02/08/2023 Shift-02

(a) $22970\sqrt{23}$

(b) $23030\sqrt{23}$

(c) $23060\sqrt{23}$

(d) $22960\sqrt{23}$

$x+\frac{1}{x}=5\sqrt{2}$

$x^3+\frac{1}{x^3}=(5\sqrt{2})^3-3\times5\sqrt{2}$

$x^3+\frac{1}{x^3}=125\times2\sqrt{2}-15\sqrt{2}$

$=235\sqrt{2}$

$x^6-\frac{1}{x^6}=\left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right)$

$=235\sqrt{2}\left(\sqrt{(235\sqrt{2})^2-4}\right)$

$=235\sqrt{2}\sqrt{55225\times2-4}$

$=235\sqrt{2}\sqrt{110446}$

$=235\sqrt{2}\sqrt{46\times2401}$

$=235\sqrt{2}\times\sqrt{2}\times\sqrt{23}\times49$

$=470\times49\times\sqrt{23}$

$=23030\sqrt{23}$

20. If $\left(x+\frac{1}{x}\right)=5\sqrt{2}$, and $x>1$, what is the value of $\left(x^6-\frac{1}{x^6}\right)$?

यदि $\left(x+\frac{1}{x}\right)=5\sqrt{2}$, और $x>1$, तो $\left(x^6-\frac{1}{x^6}\right)$ का मान क्या है?

SSC CHSL 02/08/2023 Shift-02

(a) $22970\sqrt{23}$ (b) $23030\sqrt{23}$

(c) $23060\sqrt{23}$ (d) $22960\sqrt{23}$

$x+\frac{1}{x}=5\sqrt{2}$

$x^3+\frac{1}{x^3}=(5\sqrt{2})^3-3\times5\sqrt{2}$

$=250\sqrt{2}-15\sqrt{2}$

$=235\sqrt{2}$

$x-\frac{1}{x}=\sqrt{(5\sqrt{2})^2-4}$

$=\sqrt{46}$

$x^3-\frac{1}{x^3}=(\sqrt{46})^3+3\sqrt{46}$

$=46\sqrt{46}+3\sqrt{46}$

$=49\sqrt{46}$

$x^6-\frac{1}{x^6}=\left(x^3+\frac{1}{x^3}\right)\left(x^3-\frac{1}{x^3}\right)$

$=235\sqrt{2}\times49\sqrt{46}$

$=235\times49\times\sqrt{2}\times\sqrt{2}\times\sqrt{23}$

$=470\times49\times\sqrt{23}$

$=23030\sqrt{23}$

$x^6 - \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right)\left(x^3 - \frac{1}{x^3}\right)$

$= \sqrt{(22\sqrt{2})^2 + 4} \times 22\sqrt{2}$

$= \sqrt{484 \times 2 + 4} \times 22\sqrt{2}$

$= \sqrt{972} \times 22\sqrt{2}$

$= \sqrt{3 \times 324} \times 22\sqrt{2}$

$= 18 \times \sqrt{3} \times 22\sqrt{2}$

$= 396\sqrt{6}$

21. If $\left(x - \frac{1}{x}\right) = 2\sqrt{2}$, and $x > 1$, what is the value of $\left(x^6 - \frac{1}{x^6}\right)$.

$x - \frac{1}{x} = 2\sqrt{2}$

$x^3 - \frac{1}{x^3} = (2\sqrt{2})^3 + 3 \times 2\sqrt{2}$

$= 16\sqrt{2} + 6\sqrt{2}$

$= 22\sqrt{2}$

यदि $\left(x - \frac{1}{x}\right) = 2\sqrt{2}$ और $x > 1$ है, तो $\left(x^6 - \frac{1}{x^6}\right)$ का मान क्या होगा?

SSC CGL 20/07/2023 (Shift-03)

(a) $372\sqrt{6}$

(b) $384\sqrt{6}$

(c) $396\sqrt{6}$

(d) $420\sqrt{6}$

$$x^8 - \frac{1}{x^8} = \left(x^4 + \frac{1}{x^4}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= 527 \times 23 \times 5 \times \sqrt{25-4}$$

$$= 527 \times 115 \times \sqrt{21}$$

$$= 60605\sqrt{21}$$

22. If $\left(x + \frac{1}{x}\right) = 5$, and $x > 1$, what is the value of $\left(x^8 - \frac{1}{x^8}\right)$?

यदि $\left(x + \frac{1}{x}\right) = 5$ और $x > 1$ है, तो $\left(x^8 - \frac{1}{x^8}\right)$ का मान क्या होगा?

SSC CGL 18/07/2023 (Shift-03)

(a) $60605\sqrt{21}$ ✓

(b) $60615\sqrt{21}$

(c) $60705\sqrt{21}$

(d) $60725\sqrt{21}$

$N - Q$?

$Q \longrightarrow N$?

$Q \longrightarrow Q$?

23. If $x+\frac{1}{x}=8$, then find the value of $\frac{5x}{x^2+1-6x}$.

SSC CHSL 14/10/2020 (Shift- 03)

(a) 2.5 (b) 6
(c) 5 (d) 6.5

$$\frac{\frac{5\cancel{x}}{\cancel{x}}}{\frac{x^2}{\cancel{x}}+\frac{1}{x}-\frac{6\cancel{x}}{\cancel{x}}}=\frac{5}{x+\frac{1}{x}-6}=\frac{5}{8-6}=\frac{5}{2}$$

$$\frac{\frac{7\cancel{x}}{\cancel{x}}}{\frac{x^{\cancel{2}}}{\cancel{x}}+\frac{1}{x}-\frac{8\cancel{x}}{\cancel{x}}}$$

$$= \frac{7}{x+\frac{1}{x}-8} = \frac{7}{10-8} = \frac{7}{2}$$

24. If $x+\frac{1}{x}=10$, then find the value of $\frac{7x}{x^2+1-8x}$.

SSC CHSL 03/06/2022 (Shift- 01)

(a) 3.5 (b) 4.5

(c) 2.5 (d) 5.5

25. If $x+\frac{1}{x} = 5$, $x \neq 0$ then the value of

$$\frac{x^4+\frac{1}{x^2}}{x^2-3x+1}$$ is equal to :

SSC CHSL 20/10/2020 (Shift- 01)

(a) 55 (b) 60

(c) 65 (d) 50

$$\frac{\frac{x^4}{x}+\frac{1}{x^2 \cdot x}}{\frac{x^2}{x}-\frac{3\cancel{x}}{\cancel{x}}+\frac{1}{x}}$$

$$= \frac{x^3+\frac{1}{x^3}}{x-3+\frac{1}{x}} = \frac{x^3+\frac{1}{x^3}}{x+\frac{1}{x}-3}$$

$$= \frac{5^3-3\times 5}{5-3}$$

$$= \frac{\cancel{110}}{\cancel{2}} \; 55$$

$$\frac{\frac{x^6}{x^3}+\frac{3x^3}{x^3}-\frac{1}{x^3}}{\frac{x^6}{x^3}-\frac{8x^3}{x^3}-\frac{1}{x^3}}$$

26. If $x-\frac{1}{x}=5$, $x \neq 0$, then what is the value of $\frac{x^6+3x^3-1}{x^6-8x^3-1}$?

SSC CGL TIER II 16/11/2020

$$= \frac{x^3+3-\frac{1}{x^3}}{x^3-8-\frac{1}{x^3}} = \frac{x^3-\frac{1}{x^3}+3}{x^3-\frac{1}{x^3}-8}$$

$$= \frac{\cancel{143}\ 13}{\cancel{132}\ 12}$$

(a) $\frac{13}{12}$ (b) $\frac{11}{13}$

(c) $\frac{3}{8}$ (d) $\frac{4}{9}$

$x-\frac{1}{x}=5$

$x^3-\frac{1}{x^3}=5^3+3\times5=140$

27. If $x^2 + 3x + 1 = 0$, then what is the value of $x^6 + \frac{1}{x^6}$?

SSC CGL 3/03/2020 (Shift- 02)

(a) 324
(b) 322
(c) 318
(d) 327

$\frac{x^2}{x} + \frac{3x}{x} + \frac{1}{x} = 0$

$\Rightarrow x + 3 + \frac{1}{x} = 0$

$\Rightarrow x + \frac{1}{x} = -3$

$x + \frac{1}{x} = n$

$x^6 + \frac{1}{x^6} = (n^3 - 3n)^2 - 2$

$x^6 + \frac{1}{x^6} = [-27 - 3(-3)]^2 - 2$

$= [-27 + 9]^2 - 2$

$= 324 - 2$

$= 322$

28. If $x^2 - 2\sqrt{5}x + 1 = 0$, then what is the value of $x^5 + \frac{1}{x^5}$?

SSC CGL 5/03/2020 (Shift- 01)

(a) $610\sqrt{5}$ (b) $408\sqrt{5}$

(c) $612\sqrt{5}$ (d) $406\sqrt{5}$

$$\frac{x^2}{x} - \frac{2\sqrt{5}x}{x} + \frac{1}{x} = 0$$

$$\Rightarrow x - 2\sqrt{5} + \frac{1}{x} = 0$$

$$\Rightarrow x + \frac{1}{x} = 2\sqrt{5}$$

$$x^5 + \frac{1}{x^5} = (20-2)(40\sqrt{5} - 6\sqrt{5}) - 2\sqrt{5}$$

$$= 18 \times 34\sqrt{5} - 2\sqrt{5}$$

$$= 612\sqrt{5} - 2\sqrt{5}$$

$$= 610\sqrt{5}$$

$$x + \frac{1}{x} = n$$

$$x^5 + \frac{1}{x^5} = (n^2-2)(n^3-3n) - n$$

$6 = 2x + 5 - \frac{2}{x}$

$\Rightarrow 1 = 2\left(x - \frac{1}{x}\right)$

$\Rightarrow \boxed{\frac{1}{2} = x - \frac{1}{x}}$

$\therefore x + \frac{1}{x} = \sqrt{\left(\frac{1}{2}\right)^2 + 4} = \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2}$

$x^3 + \frac{1}{x^3} = \left(\frac{\sqrt{17}}{2}\right)^3 - 3\left(\frac{\sqrt{17}}{2}\right)$

$= \frac{17\sqrt{17}}{8} - \frac{4 \times 3\sqrt{17}}{4 \times 2}$

$= \frac{5\sqrt{17}}{8}$

29. If $\frac{6x}{2x^2 + 5x - 2} = 1$, $x > 0$, then the value of $x^3 + \frac{1}{x^3}$ is :

$\frac{6}{2x + 5 - \frac{2}{x}} = 1$

SSC CGL 07/06/2019 (Shift- 03)

(a) $\frac{3}{8}\sqrt{17}$

(b) $\frac{5}{8}\sqrt{17}$

(c) $\frac{5}{16}\sqrt{17}$

(d) $\frac{3}{4}\sqrt{17}$

30. If $x^2 - 5x + 1 = 0$, then the value of

$$\frac{x^6}{x^3} + \frac{x^4}{x^3} + \frac{x^2}{x^3} + \frac{1}{x^3} = ?$$

$$\frac{5x^3}{x^3}$$

SSC CGL 01/12/2022 (Shift-01)

(a) 30 (b) 25

(c) 23 (d) 28

$$\frac{x^2}{x} - \frac{5x}{x} + \frac{1}{x} = 0$$

$$\Rightarrow x - 5 + \frac{1}{x} = 0$$

$$\Rightarrow x + \frac{1}{x} = 5$$

$$\frac{x^3 + x + \frac{1}{x} + \frac{1}{x^3}}{5} = \frac{\left(x + \frac{1}{x}\right) + \left(x^3 + \frac{1}{x^3}\right)}{5} = \frac{5 + 110}{5} = \frac{\cancel{118}\ 23}{\cancel{5}}$$

$x - 3 + \frac{1}{x} = 0$

$\Rightarrow x + \frac{1}{x} = 3$

$\frac{x^3 + \frac{1}{x^3}}{x + 5 + \frac{1}{x}} = \frac{(3)^3 - 3 \times 3}{3 + 5} = \frac{\cancel{18}\,9}{\cancel{8}\,4}$

31. If $\frac{x^2}{x} - \frac{3x}{x} + \frac{1}{x} = 0$, then the value of $\frac{\frac{x^4}{x} + \frac{1}{x^2 \cdot x}}{\frac{x^2}{x} + 5\cancel{\frac{x}{x}} + \frac{1}{x}}$ is:

SSC CGL MAINS 03/02/2022

(a) $\frac{9}{4}$ ✓

(b) $\frac{27}{8}$

(c) $\frac{5}{2}$

(d) 2

32. If $x^2 - 3x - 1 = 0$, then the value of $(x^2 + 8x - 1)(x^3 + x^{-1})^{-1}$ is :

SSC CGL 07/06/2019 (Shift- 02)

(a) $\frac{3}{8}$ (b) 8

(c) 1 (d) 3

$$\frac{x^2}{x} - \frac{3x}{x} - \frac{1}{x} = 0$$

$$\Rightarrow x - 3 - \frac{1}{x} = 0 \quad \boxed{\Rightarrow x - \frac{1}{x} = 3}$$

$$\left(\frac{x^2}{x} + \frac{8x}{x} - \frac{1}{x}\right) \times \frac{1}{\left(\frac{x^3}{x} + \frac{1}{x \cdot x}\right)^1}$$

$$= \frac{\left(x + 8 - \frac{1}{x}\right)}{\left(x^2 + \frac{1}{x^2}\right)} = \frac{3+8}{3^2+2} = \cancel{\frac{11}{11}}\ 1$$

$$\frac{a^{-1}}{1} = \frac{1}{a^1}$$

$$2^{-1} = \frac{1}{2}$$

$$x - \frac{1}{x} = n$$

$$x^2 + \frac{1}{x^2} = n^2 + 2$$

Equal Coefficient

* $x+\frac{1}{x}=3$

$x^2+\frac{1}{x^2}=3^2-2=7$

* $2x+\frac{1}{2x}=3$

$\left(2x+\frac{1}{2x}\right)^2=(3)^2$

$\Rightarrow 4x^2+\frac{1}{4x^2}+2\times\cancel{2x}\times\frac{1}{\cancel{2x}}=(3)^2$

$\Rightarrow 4x^2+\frac{1}{4x^2}=(3)^2-2\times\cancel{2x}\times\frac{1}{\cancel{2x}}$

Unequal Coefficient

* $3x+\frac{1}{2x}=4$

Soln $\left(3x+\frac{1}{2x}\right)^2=(4)^2$

$\Rightarrow 9x^2+\frac{1}{4x^2}+2\times3\cancel{x}\times\frac{1}{2\cancel{x}}=(4)^2$

$\Rightarrow 9x^2+\frac{1}{4x^2}=(4)^2-\cancel{2}\times3\times\frac{1}{\cancel{2}}$

$=13$

Q. $4x + \frac{1}{2x} = 2$

$16x^2 + \frac{1}{4x^2} = ?$

Ans

$(2)^2 - \cancel{2} \times 4 \times \frac{1}{\cancel{2}}$

$= 4 - 4$

$= \underline{\underline{0}}$

Q. $3x + \frac{1}{2x} = 4$

$27x^3 + \frac{1}{8x^3} = ?$

Ans

$(4)^3 - 3 \times \overset{2}{\cancel{4}} \times 3 \times \frac{1}{\cancel{2}}$

$= 64 - 18$

$= \underline{\underline{46}}$

Q. $3x + \frac{1}{2x} = 4$

$$3x - \frac{1}{2x} = \sqrt{(4)^2 - \cancel{4}^{2} \times 3 \times \frac{1}{\cancel{2}}}$$

$$= \sqrt{16-6}$$

$$= \underline{\underline{\sqrt{10}}}$$

Q. $4x + \frac{1}{2x} = 5$

$$4x - \frac{1}{2x} = \sqrt{(5)^2 - 4 \times \cancel{4}^{2} \times \frac{1}{\cancel{2}}}$$

$$= \sqrt{25-8} = \sqrt{17}$$

Q $3x + \frac{1}{4x} = 4$

$9x^2 + \frac{1}{16x^2} = ?$

Ans

$$9x^2 + \frac{1}{16x^2} = (4)^2 - \cancel{2} \times 3 \times \frac{1}{\cancel{4}\,2}$$

$$= 16 - \frac{3}{2}$$

$$= \frac{29}{2}$$

Q. $7x + \frac{1}{2x} = 11$

$49x^2 + \frac{1}{4x^2} = ?$

Ans

$$49x^2 + \frac{1}{4x^2} = (11)^2 - \cancel{2} \times 7 \times \frac{1}{\cancel{2}}$$

$$= 121 - 7$$

$$= 114$$

Q. $2x + \frac{1}{3x} = 4$ $\qquad 9x^2 + \frac{1}{4x^2} = ?$

$3x + \frac{1}{2x}$

Sol$^{\underline{n}}$ $\quad 2x + \frac{1}{3x} = 4$

$3x + \frac{1}{2x} = \cancel{4}^{2} \times 3 \times \frac{1}{\cancel{2}} = 6$

$9x^2 + \frac{1}{4x^2} = (6)^2 - \cancel{2} \times 3 \times \frac{1}{\cancel{2}}$

$= \underline{\underline{33}}$

$2x + \frac{1}{3x} = 4$

$\times \frac{3}{2}$

$3x + \frac{1}{2x} = 4 \times \frac{3}{2}$

$\times \frac{3}{2}$

Q. $4x + \frac{1}{3x} = 16$

$3x + \frac{1}{4x} = ?$

Ans $3x + \frac{1}{4x} = \cancel{16}^{4} \times 3 \times \frac{1}{\cancel{4}} = \underline{\underline{12}}$

33. If $5x + \frac{1}{3x} = 5$, find the value of $9x^2 + \frac{1}{25x^2}$

(a) 0 (b) 5

(c) 4 (d) 7.8

$5x + \frac{1}{3x} = 5$

$\Rightarrow 3x + \frac{1}{5x} = \cancel{5} \times 3 \times \frac{1}{\cancel{5}}$

$9x^2 + \frac{1}{25x^2} = (3)^2 - 2 \times 3 \times \frac{1}{5}$

$= 9 - 1.2$

$= 7.8$

34. If $3x + \frac{1}{2x} = 5$ then find $8x^3 + \frac{1}{27x^3}$

(a) $118\frac{1}{2}$

(b) $30\frac{10}{27}$

(c) 0

(d) 1

$3x + \frac{1}{2x} = 5$

$\Rightarrow 2x + \frac{1}{3x} = 5 \times 2 \times \frac{1}{3}$

$\Rightarrow 8x^3 + \frac{1}{27x^3} = \left(\frac{10}{3}\right)^3 - \cancel{3} \times \frac{10}{\cancel{3}} \times 2 \times \frac{1}{3}$

$= \frac{1000}{27} - \frac{20}{3}$

$= \frac{1000 - 180}{27} = \frac{820}{27} = 30\frac{10}{27}$

$x+\frac{1}{16x}=3$

35. If $x + \frac{1}{16x} = 3$, then the value of $16x^3 + \frac{1}{256x^3}$ is :

$\Rightarrow x^3+\frac{1}{(16)^3x^3}=(3)^3-3\times3\times1\times\frac{1}{16}$

$\Rightarrow x^3+\frac{1}{(16)^3x^3}=27-\frac{9}{16}$

SSC CGL TIER II 12/09/2019

(a) 423 (b) 441
(c) 432 (d) 414

$\Rightarrow 16x^3+\cancel{16}\times\frac{1}{\cancel{16}\times16\times16\times x^3}=27\times16-\frac{9}{\cancel{16}}\times\cancel{16}$

$\Rightarrow 16x^3+\frac{1}{256x^3}=432-9=423$

36. If $5x + \frac{1}{3x} = 4$, then the value of $9x^2 + \frac{1}{25x^2}$ is :

H.W

SSC CGL 04/03/2020(*Shift-01*)

(a) $\frac{174}{125}$

(b) $\frac{144}{125}$

(c) $\frac{114}{25}$

(d) $\frac{119}{25}$

37. If $\frac{20x^2}{x} - \frac{30x}{x} + \frac{1}{x} = 0$, then what is the value of $25x^2 + \frac{1}{16x^2}$ is :

SSC CGL 05/03/2020 (Shift-02)

(a) $53\frac{1}{2}$ (b) $58\frac{1}{2}$

(c) $53\frac{3}{4}$ (d) $58\frac{3}{4}$

$20x - 30 + \frac{1}{x} = 0$

$\Rightarrow \frac{20x}{4} + \frac{1}{4x} = \frac{30}{4}$

$5x + \frac{1}{4x} = \frac{15}{2}$

$25x^2 + \frac{1}{16x^2} = \left(\frac{15}{2}\right)^2 - 2 \times 5 \times \frac{1}{4}$

$= \frac{225}{4} - \frac{10}{4} = \frac{215}{4} = 53\frac{3}{4}$

38. If $4a + \frac{1}{5a} = 4$, then the value of $25a^2 + \frac{1}{16a^2}$ is :

H.W

SSC CHSL 21/10/2020 (Shift-03)

(a) $\frac{45}{2}$

(b) $\frac{55}{2}$

(c) $\frac{43}{2}$

(d) $\frac{45}{4}$

39. If $\frac{2x^2}{x} + \frac{5x}{x} + \frac{1}{x} = 0$, then one of the values of $x - \frac{1}{2x}$?

SSC CGL MAINS 29/01/2022

(a) $\frac{\sqrt{17}}{2}$ (b) $\frac{13}{2}$

(c) $\frac{5}{2}$ (d) $\frac{\sqrt{13}}{2}$

$2x + 5 + \frac{1}{x} = 0$

$\Rightarrow 2x + \frac{1}{x} = -5$

$\Rightarrow x + \frac{1}{2x} = -5 \times 1 \times \frac{1}{2} = -\frac{5}{2}$

$x - \frac{1}{2x} = \sqrt{\left(-\frac{5}{2}\right)^2 - 4 \times 1 \times \frac{1}{2}}$

$= \sqrt{\frac{25}{4} - 2}$

$= \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$

$a + \frac{1}{a} = n$

$a - \frac{1}{a} = \sqrt{n^2 - 4}$

40. If $5x - \frac{1}{4x} = 6$, $x > 0$, then find the value of $25x^2 - \frac{1}{16x^2}$.

SSC CGL 21/04/2022 (Shift 01)

(a) $6\sqrt{41}$ (b) 36

(c) $\sqrt{246}$ (d) $6\sqrt{31}$

$25x^2 - \frac{1}{16x^2}$

$= (5x)^2 - \frac{1}{(4x)^2}$

$= \left(5x - \frac{1}{4x}\right)\left(5x + \frac{1}{4x}\right)$

$= 6\sqrt{(6)^2 + \cancel{4} \times 5 \times \frac{1}{\cancel{4}}}$

$= 6\sqrt{36+5}$

$= 6\sqrt{41}$

$(2x)^2 + \frac{1}{(2x)^2} = 2$

$t^2 + \frac{1}{t^2} = 2$

$t^3 + \frac{1}{t^3} = ?$

41. If $\left(4x^2 + \frac{1}{4x^2}\right) = 2$, than what is the value of $\left(8x^3 + \frac{1}{8x^3}\right)$?

$(2x)^3 + \frac{1}{(2x)^3}$

(a) +2 (b) –2

(c) +4 (d) –4

let $2x = t$

$2x = 1$

$x = \frac{1}{2}$

$\cancel{4}x\frac{1}{\cancel{4}} + \frac{1}{\cancel{4}x\frac{1}{\cancel{4}}} = 2$

$\cancel{8}x\frac{1}{\cancel{8}} + \frac{1}{\cancel{8}x\frac{1}{\cancel{8}}} = 2$

41. If $\left(4x^2 + \frac{1}{4x^2}\right) = 2$, than what is the value of $\left(8x^3 + \frac{1}{8x^3}\right)$?

$(2x)^3 + \frac{1}{(2x)^3}$

(a) +2 (b) –2

(c) +4 (d) –4

let $2x = t$

$2x = 1$

$x = \frac{1}{2}$

42. If $\left(0.4x + \frac{1}{x}\right) = 5$, what is the value of

$\left(0.064x^3 + \frac{1}{x^3}\right)$?

$= (5)^3 - 3 \times \cancel{5} \times \frac{0.\cancel{4}^{2}}{\cancel{10}\,\cancel{2}} \times 1$

$= 125 - 6 = 119$

SSC CGL 20/04/2022 (Shift 01)

(a) 119 (b) 125

(c) 110 (d) 105

$(0.4)^2 = 0.16$

$(0.4)^3 = 0.064$

Q. $x^2+\frac{1}{x^2}=119$

$x+\frac{1}{x}=?$

Ans

$x^2+\frac{1}{x^2}=119$

$x+\frac{1}{x}=\sqrt{119+2}=11$

Q. $x^4+\frac{1}{x^4}=119$

$x+\frac{1}{x}=?$

Ans

$x^4+\frac{1}{x^4}=119$

$x^2+\frac{1}{x^2}=\sqrt{119+2}=11$

$x+\frac{1}{x}=\sqrt{11+2}=\sqrt{13}$

$\sqrt{x}+\frac{1}{\sqrt{x}}=\sqrt{\sqrt{13}+2}$ ✓

Q. $x^4+\frac{1}{x^4}=119$ find $x+\frac{1}{x}$

$x-\frac{1}{x}$

Sol^n $x^4+\frac{1}{x^4}=119$

$x^2+\frac{1}{x^2}=\sqrt{119+2}=11$

$x+\frac{1}{x}=\sqrt{11+2}$

$=\sqrt{13}$.

$x-\frac{1}{x}=\sqrt{11-2}=\underline{\underline{3}}$

$x^2+\frac{1}{x^2}=27$

$x+\frac{1}{x}=\sqrt{27+2}$

$=\sqrt{29}$

$x-\frac{1}{x}=\sqrt{27-2}$

$=\sqrt{25}=5$

43. If $x^2+\frac{1}{x^2}=27$, then find

(i) $x+\frac{1}{x}$ (ii) $x-\frac{1}{x}$

44. If $x^4 + \frac{1}{x^4} = 23$, find

(i) $x + \frac{1}{x}$ (ii) $x^3 + \frac{1}{x^3}$

$x^4 + \frac{1}{x^4} = 23$

$x^2 + \frac{1}{x^2} = \sqrt{23+2} = 5$

* $x + \frac{1}{x} = \sqrt{5+2} = \sqrt{7}$

* $x^3 + \frac{1}{x^3} = (\sqrt{7})^3 - 3\sqrt{7}$

$= 7\sqrt{7} - 3\sqrt{7} = 4\sqrt{7}$

$x^4 - 1442 + \frac{1}{x^4} = 0$

$\Rightarrow x^4 + \frac{1}{x^4} = 1442$

$x^2 + \frac{1}{x^2} = \sqrt{1442+2} = 38$

$x - \frac{1}{x} = \sqrt{38-2} = 6$

45. If $\frac{x^8}{x^4} - 1442\frac{x^4}{x^4} + \frac{1}{x^4} = 0$, then a possible value of $x - \frac{1}{x}$ is :

SSC CGL TIER-II (11/09/2019)

(a) 5
(b) 8
(c) 4
(d) 6

46. If $x > 0$ and $x^4 + \frac{1}{x^4} = 142$, what is the value of $x^7 + \frac{1}{x^7}$?

SSC CGL 20/07/2023 (Shift-02)

(a) $1561\sqrt{14}$ (b) $1563\sqrt{14}$

(c) $1560\sqrt{14}$ (d) $1562\sqrt{14}$

$x^4 + \frac{1}{x^4} = 142$

$x^2 + \frac{1}{x^2} = \sqrt{142+2} = 12$

$x + \frac{1}{x} = \sqrt{12+2} = \sqrt{14}$

$x^7 + \frac{1}{x^7} = (n^3 - 3n) \times 142 - \sqrt{14}$

$= (14\sqrt{14} - 3\sqrt{14}) \times 142 - \sqrt{14}$

$= 11\sqrt{14} \times 142 - \sqrt{14}$

$= 1562\sqrt{14} - \sqrt{14}$

$= 1561\sqrt{14}$

$x^7 + \frac{1}{x^7} = \left(x^3 + \frac{1}{x^3}\right)\left(x^4 + \frac{1}{x^4}\right) - \left(x + \frac{1}{x}\right)$

47. If $x > 0$, and $x^4 + \frac{1}{x^4} = 2207$, what is the value of $x^7 + \frac{1}{x^7}$?

H.W

SSC CGL 02/12/2022 (Shift-01)

(a) 710649
(b) 710647
(c) 710654
(d) 710661

48. If $x^4 + x^{-4} = 47$, $(x > 0)$, then the value of $(2x - 3)^2$ is :

SSC CHSL 8/07/2019 (Shift-03)

(a) 2
(b) 3
(c) 5
(d) 4

$x^4 + \frac{1}{x^4} = 47$

$x^2 + \frac{1}{x^2} = \sqrt{47+2} = 7$

$x + \frac{1}{x} = \sqrt{7+2} = 3$

$\Rightarrow x^2 + 1 = 3x$

$\Rightarrow \boxed{x^2 - 3x = -1}$

$4x^2 + 9 - 2 \times 2x \times 3$

$= 4x^2 + 9 - 12x$

$= 4x^2 - 12x + 9$

$= 4(x^2 - 3x) + 9$

$= 4(-1) + 9$

$= -4 + 9 = 5$

49. If $x^4 + x^{-4} = 194$, $(x > 0)$, then the value of $(2x - 4)^2$ is :

SSC CHSL 09/07/2019 (Shift-01)

(a) 15 (b) 20
(c) 12 (d) 16

$x^4 + \frac{1}{x^4} = 194$

$x^2 + \frac{1}{x^2} = \sqrt{194+2} = 14$

$\Rightarrow x + \frac{1}{x} = \sqrt{14+2} = 4$

$\Rightarrow x^2 + 1 = 4x$

$\Rightarrow x^2 - 4x = -1$

$4x^2 + 16 - 2 \times 2x \times 4$

$= 4x^2 - 16x + 16$

$= 4(x^2 - 4x) + 16$

$= 4(-1) + 16$

$= 12$

49. If $x^4 + x^{-4} = 194$, $(x > 0)$, then the value of $(2x - 4)^2$ is :

SSC CHSL 09/07/2019 (Shift-01)

(a) 15 (b) 20
(c) 12 (d) 16

$x^4 + \frac{1}{x^4} = 194$

$x^2 + \frac{1}{x^2} = \sqrt{194+2} = 14$

$\Rightarrow x + \frac{1}{x} = \sqrt{14+2} = 4$

$x - \frac{1}{x} = \sqrt{4^2 - 4} = 2\sqrt{3}$

$x + \cancel{\frac{1}{x}} = 4$

$(+)\ x - \cancel{\frac{1}{x}} = 2\sqrt{3}$

$2x = (4 + 2\sqrt{3})$

$(2x-4)^2 = (2\sqrt{3})^2$

$\Rightarrow (2x-4)^2 = 12$

$$x = \sqrt{9}$$

$$\boxed{x = 3}$$

$$x^2 = 9$$

$$\Rightarrow x = \pm\sqrt{9}$$

$$\Rightarrow x = \pm 3$$

50. If $x^4 + \frac{1}{x^4} = 322$, find $x^3 - \frac{1}{x^3}$

(a) ± 76 (b) 76

(c) -76 (d) 95

$x^4 + \frac{1}{x^4} = 322$

$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{322+2} = 18$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 18 - 2$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 16$

$\Rightarrow x - \frac{1}{x} = \pm\sqrt{16}$

$\Rightarrow \boxed{x - \frac{1}{x} = \pm 4}$

$x - \frac{1}{x} = 4$

$x^3 - \frac{1}{x^3} = 4^3 + 3 \times 4$

$= 76$

$x - \frac{1}{x} = -4$

$x^3 - \frac{1}{x^3} = (-4)^3 + 3 \times (-4)$

$= -64 - 12$

$= -76$

Special Type

$$x + \frac{1}{x} = 2$$

$\therefore x = 1$

$$x + \frac{1}{x} = -2$$

$\therefore x = -1$

Q. $x + \frac{1}{x} = 2$ find $x^{99} + \frac{1}{x^{100}} = ?$

Ans $x = 1$

$\therefore (1)^{99} + \frac{1}{(1)^{100}}$

$= 1 + \frac{1}{1}$

$= \underline{\underline{2}}$

51. If $x + \frac{1}{x} = 2$, find

(i) $x^{20} + x^{19} + x^{18} + \ldots\ldots\ldots + x^2 + x + 1$

(ii) $x^{49} - x^{48} + x^{47} - x^{46} + \ldots\ldots + x^3 - x^2 + x + 1$

(iii) $x^{12} + \frac{1}{x^{12}}$

(iv) $x^{17} + \frac{1}{x^9}$

(v) $x^{15} - \frac{1}{x^{17}}$

51. If $x + \frac{1}{x} = 2$, find

(i) $x^{20} + x^{19} + x^{18} + \ldots\ldots + x^2 + x^1 + 1$

$= 1 + 1 + 1 + \ldots + 1 + 1 + 1$

$= 20 + 1$

$= 21$

51. If $x + \frac{1}{x} = 2$, find

(ii) $\cancel{x^{49}} - \cancel{x^{48}} + \cancel{x^{47}} - \cancel{x^{46}} + \ldots\ldots + \cancel{x^3} - \cancel{x^2} + x + 1$

$= \cancel{+1} \cancel{-1} \cancel{+1} - 1 + \ldots\ldots \cancel{+1} \cancel{-1} + 1 + 1$

$= \underline{\underline{2}}$

51. If $x + \frac{1}{x} = 2$, find

(iii) $x^{12} + \frac{1}{x^{12}}$

$= (1)^{12} + \frac{1}{(1)^{12}}$

$= 1 + 1$

$= 2$

51. If $x + \frac{1}{x} = 2$, find

(iv) $x^{17} + \frac{1}{x^9}$

$= (1)^{17} + (1)^9$

$= \underline{\underline{2}}$

51. If $x + \frac{1}{x} = 2$, find

(v) $x^{15} - \frac{1}{x^{17}}$

$= 1 - 1$

$= 0$

52. If $p + \left(\frac{1}{p}\right) = 2$, then find the value of $p \times p \times p$.

1 x 1 x 1

SSC CHSL 18 /03/2020 (Shift-02)

(a) 4
(b) 8
(c) 1
(d) 2

① $x + \frac{1}{x} = -2$ $\quad \therefore x = -1$

⊙ $x^{10} + \frac{1}{x^{10}} = 1 + 1 = 2$

⊙ $x^{9} + \frac{1}{x^{9}} = -1 - 1 = -2$

⊙ $x^{9} + \frac{1}{x^{10}} = -1 + 1 = 0$

$(-1)^{\text{even}} = 1$

$(-1)^{\text{odd}} = -1$

53. If $x + \frac{1}{x} = -2$

(i) $x^{39} + x^{38} + \ldots\ldots\ldots + x^2 + x + 1$

(ii) $x^{100} - x^{99} + x^{98} - x^{97} + \ldots\ldots + x^2 - x + 1$

(iii) $x^5 - \frac{1}{x^5}$

(iv) $x^{18} + \frac{1}{x^{18}}$

(v) $x^7 - \frac{1}{x^9}$

53. If $x + \frac{1}{x} = -2$

(i) $x^{39} + x^{38} + \ldots\ldots x^3 + x^2 + x + 1$

$= -1 + 1 - 1 + 1 \ldots - -1 + 1 - 1 + 1$

$= 0$

53. If $x + \frac{1}{x} = -2$

(ii) $x^{100} - x^{99} + x^{98} - x^{97} + \ldots\ldots + x^2 - x^1 + 1$

$= 1-(-1) + 1-(-1) + \ldots + 1-(-1)+1$

$= 1+1+1+1+\ldots+1+1+1$

$= 100+1$

$= \underline{\underline{101}}$

$x = -1$

53. If $x + \frac{1}{x} = -2$

(iii) $x^5 - \frac{1}{x^5}$

$(-1)^5 - \frac{1}{(-1)^5}$

$= -1 - \frac{1}{(-1)}$

$= -1 + 1$

$= 0$

53. If $x + \frac{1}{x} = -2$

(iv) $x^{18} + \frac{1}{x^{18}}$

$= (-1)^{18} + \frac{1}{(-1)^{18}}$

$= 1 + 1$

$= 2$

53. If $x + \frac{1}{x} = -2$

(v) $x^7 - \frac{1}{x^9}$

$= (-1)^7 - \frac{1}{(-1)^9}$

$= -1 - \frac{1}{(-1)}$

$= -1 + 1$

$= 0$

* $x+\frac{1}{x}=2$, $x=1$

* $x+\frac{1}{x}=-2$, $x=-1$

* $\boxed{x+\frac{1}{x}=1}$ $\therefore$ $x^3=-1$

* $x+\frac{1}{x}=-1$ $x^3=1$

$$* \quad x + \frac{1}{x} = 1$$

$$\Rightarrow \quad x^2 + 1 = x$$

$$\Rightarrow \quad (x^2 + 1 - x) = 0$$

$$\Rightarrow \quad (x+1)(x^2+1-x) = 0$$

$$\Rightarrow \quad (x^3+1) = 0$$

$$\Rightarrow \quad x^3 = -1$$

$$(a^3 + b^3) = (a+b)(a^2+b^2-ab)$$

Q. $x + \frac{1}{x} = 1$ find $x^{99} + \frac{1}{x^{33}}$

Ans $\boxed{x^3 = -1}$

$$(x^3)^{33} + \frac{1}{(x^3)^{11}}$$

$$= (-1)^{33} + \frac{1}{(-1)^{11}}$$

$$= -1 - 1$$

$$= -2$$

54. If $x + \frac{1}{x} = 1$, find

(i) $x^{18} + x^{12} + x^{6} + 1$

(ii) $x^{99} + x^{96} + x^{93} + x^{90} + \ldots\ldots + x^{6} + x^{3} + 1$

(iii) $x^{45} - x^{42} + x^{39} - x^{36} + \ldots\ldots + x^{9} - x^{6} + x^{3} + 1$

(iv) $x^{32} + x^{46} + x^{90} + 2$

54. If $x + \frac{1}{x} = 1$, find

(i) $x^{18} + x^{12} + x^{6} + 1$

$\therefore x + \frac{1}{x} = 1$

$\therefore x^3 = -1$

$= (x^3)^6 + (x^3)^4 + (x^3)^2 + 1$

$= (-1)^6 + (-1)^4 + (-1)^2 + 1$

$= 1 + 1 + 1 + 1$

$= 4$

$* \quad x + \frac{1}{x} = 1$ find $x^{100} + x^{103} + x^{201} + x^{204}$

Ans $\boxed{x^3 = -1}$

$$\therefore \; x^{100} + x^{103} + x^{201} + x^{204}$$

$$= x^{100}(1 + x^3) + x^{201}(1 + x^3)$$

$$= x^{100}(1 - 1) + x^{201}(1 - 1)$$

$$= \underline{0 + 0}$$

$* \quad x + \frac{1}{x} = 1$

find $\cancel{x^{100}} + \cancel{x^{103}} + \cancel{x^{201}} + \cancel{x^{204}}$

0

Ans $\boxed{x^3 = -1}$

54. If $x + \frac{1}{x} = 1$, find

(ii) $\cancel{x^{99}} + \cancel{x^{96}} + \cancel{x^{93}} + \cancel{x^{90}} + \ldots\ldots \cancel{x^{9}} + \cancel{x^{6}} + x^3 + 1$

$= -1 + 1$

$= 0$

Ans $x^3 = -1$

$\therefore$

54. If $x + \frac{1}{x} = 1$, find

(iii) $x^{45} - x^{42} + x^{39} - x^{36} + \ldots\ldots + x^{9} - x^{6} + x^{3} + 1$

$= (x^3)^{15} - (x^3)^{14} + (x^3)^{13} - (x^3)^{12} + \ldots + (x^3)^3 - (x^3)^2 + x^3 + 1$

$= (-1)^{15} - (-1)^{14} + (-1)^{13} - (-1)^{12} + \ldots + (-1)^3 - (-1)^2 + (-1) + 1$

$= -1 - 1 - 1 - 1 - 1 - 1 \ldots - 1 + 1$

$= -15 + 1$

$= -14$

$x^3 = -1$

54. If $x + \frac{1}{x} = 1$, find

(iv) $x^{32} + x^{46} + x^{90} + 2$

$= -\left(x + \frac{1}{x}\right) + 3$

$= -1 + 3$

$= 2$

$x^{32} \cdot \frac{x^1}{x^1} + x^1 \cdot x^{45} + x^{90} + 2$

$= \frac{x^{33}}{x} + x \cdot x^{45} + x^{90} + 2$

$= \frac{(x^3)^{11}}{x} + x \cdot (x^3)^{15} + (x^3)^{30} + 2$

$= \frac{-1}{x} - x + 1 + 2$

* $x+\frac{1}{x}=2 \quad \therefore \quad x=1$

* $x+\frac{1}{x}=-2 \quad \therefore \quad x=-1$

* $x+\frac{1}{x}=1 \qquad x^3=-1$

* $x+\frac{1}{x}=-1 \qquad x^3=1$

Special case

$$x+\frac{1}{x}=\sqrt{3}$$

Proof:

$$x+\frac{1}{x}=\sqrt{3}$$

$$\Rightarrow x^3+\frac{1}{x^3}=(\sqrt{3})^3-3\sqrt{3}$$

$$\Rightarrow x^3+\frac{1}{x^3}=3\sqrt{3}-3\sqrt{3}$$

$$\Rightarrow x^3+\frac{1}{x^3}=0$$

$$\Rightarrow x^3+\frac{1}{x^3}=0 \Rightarrow \frac{x^3\cdot x^3+1}{x^3}=0$$

$$\Rightarrow \frac{x^6+1}{x^3}=0$$

$$\Rightarrow x^6+1=0$$

$$\Rightarrow x^6=-1$$

* $x+\frac{1}{x}=\sqrt{3}$ $\quad\therefore x^6=-1$

* $x+\frac{1}{x}=-\sqrt{3}$ $\quad x^6=-1$

<u>proof</u>: $x+\frac{1}{x}=-\sqrt{3}$

$\Rightarrow x^3+\frac{1}{x^3}=(-\sqrt{3})^3-3(-\sqrt{3})$

$\Rightarrow x^3+\frac{1}{x^3}=\cancel{-3\sqrt{3}}+\cancel{3\sqrt{3}}$

$\Rightarrow x^3+\frac{1}{x^3}=0$

$x^6+1=0$

$\therefore x^6=-1$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(i) $x^{90} + \frac{1}{x^{90}}$ (ii) $x^{92} + \frac{1}{x^{92}}$

(iii) $x^{102} + x^{96} + x^{101} + x^{95} + x^{100} + x^{94}$ is

(iv) $x^6 - \frac{1}{x^6} + 2$ (v) $x^{18} + x^{12} + x^6 + 1$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(i) $x^{90} + \frac{1}{x^{90}}$

$x^6 = -1$

$(x^6)^{15} + \frac{1}{(x^6)^{15}}$

$= (-1)^{15} + \frac{1}{(-1)^{15}}$

$= -1 - 1$

$= -2$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(ii) $x^{92} + \frac{1}{x^{92}}$

$x^6 = -1$

$x^2 \cdot x^{90} + \frac{1}{x^2 \cdot x^{90}}$

$= x^2 \cdot (x^6)^{15} + \frac{1}{x^2 \cdot (x^6)^{15}}$

$= x^2 (-1)^{15} + \frac{1}{x^2 (-1)^{15}}$

$= -x^2 - \frac{1}{x^2}$

$= -\left(x^2 + \frac{1}{x^2}\right)$

$= -\{(\sqrt{3})^2 - 2\}$

$= -\{3-2\}$

$= -1$

Q. $x+\frac{1}{x}=\sqrt{3}$

(a) $x^{63}+\frac{1}{x^{63}}$

$= x^3 \cdot x^{60} + \frac{1}{x^3 \cdot x^{60}}$

$= x^3 (x^6)^{10} + \frac{1}{x^3 (x^6)^{10}}$

$= x^3 (-1)^{10} + \frac{1}{x^3 (-1)^{10}}$

$= x^3 + \frac{1}{x^3}$

$= (n^3 - 3n)$

$= (\sqrt{3})^3 - 3\sqrt{3}$

$= 3\sqrt{3} - 3\sqrt{3}$

$= \underline{\underline{0}}$

Q. $x+\frac{1}{x}=\sqrt{3}$

Ans $x^6=-1$

find $x^{100}+x^{106}+x^{200}+x^{206}$

$=x^{100}(1+x^6)+x^{200}(1+x^6)$

$=x^{100}(1-1)+x^{200}(1-1)$

$=0+0$

$=0$

$x^6 = -1$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(iii) $x^{102} + x^{96} + x^{101} + x^{95} + x^{100} + x^{94}$ is

$x^6 = -1$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

(iv) $x^6 - \frac{1}{x^6} + 2$

Ans $(-1) - \frac{1}{(-1)} + 2$

$= \cancel{-1} \cancel{+1} + 2$

$= 2$

55. If $x + \frac{1}{x} = \sqrt{3}$, find

$x^6 = -1$

(v) $x^{18} + x^{12} + x^6 + 1$

$(x^6)^3 + (x^6)^2 + x^6 + 1$

$= (-1)^3 + (-1)^2 - 1 + 1$

$= \cancel{-1} + \cancel{1} \cancel{-1} + \cancel{1}$

$= 0$

56. If $\left(x+\frac{1}{x}\right)^2 = 3$, then what is the value of $x^6 + x^{-6}$?

SSC CGL MAINS (08/08/2022)

(a) 6
(b) 2
(c) –2
(d) –6

$x+\frac{1}{x}=\sqrt{3}$

$= x^6+\frac{1}{x^6}$

$= -1-1$

$= -2$

57. If $x + \frac{1}{x} = \sqrt{3}$, then the value of $x^{18} + x^{12} + x^{6} + 1$ is :

SSC CHSL 21/10/2020 (Shift- 03)

(a) 0 (b) 2

(c) 3 (d) 1

$= -1 + 1$

$= 0$

58. If $x + \frac{1}{x} = -\sqrt{3}$ then $x^{67} + x^{53} + x^{43} + x^{29} + x^{24} + x^{12} + x^{6} + 3$ is

(a) $\sqrt{3}$ (b) 0

(c) $2(2+\sqrt{3})$ (d) $2(2-\sqrt{3})$

$x^{67}+x^{53}+x^{43}+x^{29}+x^{24}+3$

$= x\cdot x^{66}+x^{53}\cdot\frac{x}{x}+x^{1}\cdot x^{42}+x^{29}\cdot\frac{x}{x}+(x^{6})^{4}+3$

$= x\cdot(x^{6})^{11}+\frac{(x^{6})^{9}}{x}+x\cdot(x^{6})^{7}+\frac{(x^{6})^{5}}{x}+(x^{6})^{4}+3$

$= x(-1)-\frac{1}{x}+x(-1)+\frac{(-1)}{x}+1+3$

$= -x-\frac{1}{x}-x-\frac{1}{x}+4$

$= -2x-\frac{2}{x}+4$

$= -2\left(x+\frac{1}{x}\right)+4$

$= -2(-\sqrt{3})+4$

$= 2\sqrt{3}+4$

$= 2(\sqrt{3}+2)$

59. If $x = 2 + \sqrt{3}$, find $x + \frac{1}{x}$

(a) 5 (b) 4

(c) –4 (d) $2\sqrt{3}$

ans $= x + \frac{1}{x}$

$= (2+\sqrt{3}) + \frac{1}{(2+\sqrt{3})}$

$= 2+\sqrt{3} + \frac{1}{(2+\sqrt{3})} \times \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$

$= 2+\sqrt{3} + \frac{(2-\sqrt{3})}{4-3}$

$= 2+\cancel{\sqrt{3}} + \frac{2-\cancel{\sqrt{3}}}{1}$

$= 4$

Note:

$$x = \overset{B}{2} + \overset{S}{\sqrt{3}}$$

If $B^2 - S^2 = 1$

$x + \frac{1}{x} = 2 \times B = 2 \times 2 = 4$

$x - \frac{1}{x} = 2 \times S = 2 \times \sqrt{3} = 2\sqrt{3}$

59. If $x = 2 + \sqrt{3}$, find $x + \frac{1}{x}$

(a) 5 (b) 4

(c) –4 (d) $2\sqrt{3}$

$\therefore x + \frac{1}{x} = 2 \times 2 = 4$

60. If $x = 7 + 4\sqrt{3}$ then $x - \frac{1}{x}$

B S

(a) $8\sqrt{3}$ (b) 4

(c) -4 (d) $-2\sqrt{3}$

$\therefore x + \frac{1}{x} = 2 \times 7 = 14$

$x - \frac{1}{x} = 2 \times 4\sqrt{3} = 8\sqrt{3}$

61. If $x = 2 + \sqrt{5}$ then the value of $x^3 - x^{-3}$ is :

SSC CHSL 08/07/2019 (Shift-03)

(a) – 52
(b) 52
(c) 76
(d) – 76

$x = \overset{S}{2} + \overset{B}{\sqrt{5}}$

$x^3 - \frac{1}{x^3}$

$x - \frac{1}{x} = 2 \times 2 = 4$

$x^3 - \frac{1}{x^3} = (4)^3 + 3 \times 4 = \underline{\underline{76}}$

M-1

62. If $x = 3 + 2\sqrt{2}$, then the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$ is:

SSC CHSL 26/10/2020 (Shift-03)

(a) 2 (b) 1
(c) 0 (d) 3

$x = 3+2\sqrt{2}$

$\therefore \sqrt{x} = \sqrt{3+2\sqrt{2}}$

$= \sqrt{2+1+2\sqrt{2}}$

$= \sqrt{(\sqrt{2})^2+(1)^2+2\times\sqrt{2}\times 1}$

$\sqrt{x} = \sqrt{(\sqrt{2}+1)^2}$

$\therefore \sqrt{x} = \overset{B}{\sqrt{2}} + \overset{S}{1}$

$\sqrt{x} - \frac{1}{\sqrt{x}} = 2\times 1 = 2$

M-1

$x = 3+2\sqrt{2}$

$\therefore x + \frac{1}{x} = 6$

$\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{6+2}$

$\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{6-2}$

$= 2$

62. If $x = 3 + 2\sqrt{2}$, then the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$ is:

SSC CHSL 26/10/2020 (Shift-03)

(a) 2
(b) 1
(c) 0
(d) 3

63. If $x = 1 + \sqrt{2}$, then the value of $\sqrt{x} + \left(\frac{1}{\sqrt{x}}\right)$ is:

SSC CHSL 17/03/2020 (Shift-03)

(a) 2.1014 (b) 2.1973
(c) 1.9876 (d) 1.9996

$x = \overset{S}{1} + \overset{B}{\sqrt{2}}$

$\therefore x + \frac{1}{x} = 2\sqrt{2}$

$\downarrow$

$\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{2\sqrt{2}+2} = \sqrt{2 \times 1.414 + 2} = \sqrt{4.82}$

$(21)^2 = 441$
$(22)^2 = 484$

64. If $p = 7 + 4\sqrt{3}$, then what is the value of $\frac{p^6 + p^4 + p^2 + 1}{p^3}$?

B S

SSC CGL 06/12/2022 (Shift-02)

(a) 2617 (b) 2167

(c) 2716 (d) 2176

$p + \frac{1}{p} = 14$

$\left(p + \frac{1}{p}\right) + \left(p^3 + \frac{1}{p^3}\right)$

$= 14 + (14)^3 - 3 \times 14$

$= 2744 - 28$

$= \underline{2716}$

$p^3 + p + \frac{1}{p} + \frac{1}{p^3}$

65. If $P = 7 + 4\sqrt{3}$ and $PQ = 1$, then what is the value of $\left(\frac{1}{P^2} + \frac{1}{Q^2}\right) = ?$

(a) 148 (b) 189

(c) 194 (d) 204

$PQ = 1$

$P = \frac{1}{Q}$

$\therefore P + \frac{1}{P} = 14$

$P^2 + \frac{1}{P^2} = (14)^2 - 2$

$= 194$

$\frac{1}{P^2} + P^2$

66. If $x = \frac{\sqrt{5}-\sqrt{4}}{\sqrt{5}+\sqrt{4}}$ and $y = \frac{\sqrt{5}+\sqrt{4}}{\sqrt{5}-\sqrt{4}}$ then the value of $\frac{x^2-xy+y^2}{x^2+xy+y^2} = ?$

SSC CGL 08/12/2022 (Shift-02)

(a) $\frac{361}{363}$ (b) $\frac{341}{343}$

(c) $\frac{384}{387}$ (d) $\frac{321}{323}$

$x = \frac{(\sqrt{5}-\sqrt{4})}{\sqrt{5}+\sqrt{4}} \times \frac{(\sqrt{5}-\sqrt{4})}{\sqrt{5}-\sqrt{4}}$

$= \frac{(\sqrt{5}-\sqrt{4})^2}{5-4} = \frac{5+4-2\times\sqrt{5}\times\sqrt{4}}{1}$

$\boxed{x = 9-4\sqrt{5}}$ $\therefore x+\frac{1}{x} = 18$

$y = \frac{1}{x}$

$\frac{x^2-\cancel{x}\times\frac{1}{\cancel{x}}+\frac{1}{x^2}}{x^2+\cancel{x}\times\frac{1}{\cancel{x}}+\frac{1}{x^2}} = \frac{x^2+\frac{1}{x^2}-1}{x^2+\frac{1}{x^2}+1}$

$= \frac{(18)^2-2-1}{(18)^2-2+1} = \frac{321}{323}$

67. If $m + \frac{1}{m-3} = 5$, then $(m-3)^8 + \frac{1}{(m-3)^8} = ?$

(a) 2 (b) 0

(c) –2 (d) 15

let $(m-3) = x$

$m + \frac{1}{(m-3)} = 5$

$\Rightarrow m + \frac{1}{(m-3)} - 3 = 5 - 3$

$\Rightarrow (m-3) + \frac{1}{(m-3)} = 2$

$x + \frac{1}{x} = 2$

$x^8 + \frac{1}{x^8} = 2$

68. If $x+\frac{1}{x-9}=11$, then $(x-11)^{12}+\frac{1}{(x-11)^{13}}=?$

(a) 0 (b) 2

(c) –2 (d) 1

$x+\frac{1}{x-9}=11$

$\Rightarrow (x-11)=-\frac{1}{(x-9)}$

$\Rightarrow (x-9)=-\frac{1}{(x-11)}$

$\Rightarrow x-9-2=\frac{-1}{(x-11)}-2$

$\Rightarrow (x-11)+\frac{1}{(x-11)}=-2$

$(x-11)^{12}+\frac{1}{(x-11)^{13}}$

$=(-1)^{12}+\frac{1}{(-1)^{13}}=1-1=0$

$(x-11)^{12}+\frac{1}{(x-11)^{13}}$

$(x-11)+\frac{1}{(x-11)}$

69. If $a + \frac{1}{a-4} = 6$, then $(a-3)^7 + \frac{1}{(a-7)^3} = ?$

(a) $63\frac{7}{8}$

(b) $255\frac{1}{8}$

(c) $127\frac{7}{8}$

(d) 216

$\frac{1}{a-4} = 6-a$

$\Rightarrow 1 = 6a - a^2 - 24 + 4a$

$\Rightarrow a^2 - 10a + 25 = 0$

$\Rightarrow a^2 - 5a - 5a + 25 = 0$

$\Rightarrow a(a-5) - 5(a-5) = 0$

$\Rightarrow (a-5)(a-5) = 0$

$a = 5$

$(a-3) + \frac{1}{(a-7)^3}$

$2^7 + \left(\frac{1}{-2}\right)^3$

$= 128 - \frac{1}{8} = 127 + 1 - \frac{1}{8}$

$= 127\frac{7}{8}$

69. If $a + \frac{1}{a-4} = 6$, then $(a-3)^7 + \frac{1}{(a-7)^3} = ?$

(a) $63\frac{7}{8}$

(b) $255\frac{1}{8}$

(c) $127\frac{7}{8}$

(d) 216

$(a-4) + \frac{1}{(a-4)} = 2$

$\therefore a-4 = 1$

$a = 5$

$2^7 + \frac{1}{(-2)^3}$

$= 128 - \frac{1}{8}$

$= 127\frac{7}{8}$

$(a-3) + \frac{1}{(a-7)^3}$

70. If $x^2 - 22x + 111 = 0$, then what is the value of $(x-8)^2 - \frac{1}{(x-8)^2} = ?$

यदि $x^2 - 22x + 111 = 0$ तो $(x-8)^2 - \frac{1}{(x-8)^2} = ?$

(a) $12\sqrt{10}$ (b) $8\sqrt{5}$

(c) $8\sqrt{3}$ (d) 18

M-1 (Basic)

$x - 8 = t$

$\Rightarrow x = 8 + t$

$\therefore (8+t)^2 - 22(8+t) + 111 = 0$

$\Rightarrow 64 + t^2 + 16t - 176 - 22t + 111 = 0$

$\Rightarrow t^2 - 6t - 1 = 0$

$\Rightarrow t - 6 - \frac{1}{t} = 0$

$\Rightarrow t - \frac{1}{t} = 6$

$t^2 - \frac{1}{t^2} = \left(t - \frac{1}{t}\right)\left(t + \frac{1}{t}\right)$

$= 6\sqrt{6^2 + 4}$

$= 6 \times 2\sqrt{10}$

$= 12\sqrt{10}$

70. If $x^2 - 22x + 111 = 0$, then what is the value of $(x-8)^2 - \frac{1}{(x-8)^2} = ?$

t

$$\begin{array}{r} 22 \\ -\ 16 \\ \hline 6 \end{array}$$

यदि $x^2 - 22x + 111 = 0$ तो $(x-8)^2 - \frac{1}{(x-8)^2} = ?$

(a) $12\sqrt{10}$ (b) $8\sqrt{5}$

(c) $8\sqrt{3}$ (d) 18

$$t^2 - \frac{1}{t^2} = \left(t - \frac{1}{t}\right)\left(t + \frac{1}{t}\right)$$

$$= 6\sqrt{6^2+4}$$

$$= 6 \times 2\sqrt{10}$$

$$= 12\sqrt{10}$$

71. If $x^2 + 13x + 39 = 0$, then $(x+8)^5 - \frac{1}{(x+8)^5}$ = ?

(a) 393 (b) 396

(c) 392 (d) 394

let $x+8=t$

$x = t-8$

$\therefore (t-8)^2+13(t-8)+39=0$

$\Rightarrow t^2+64-16t+13t-104+39=0$

$\Rightarrow t^2-3t+103-104=0$

$\Rightarrow t^2-3t-1=0$

$\Rightarrow t-3-\frac{1}{t}=0$

$\Rightarrow t-\frac{1}{t}=3$

$t^5-\frac{1}{t^5} = (n^2+2)(n^3+3n)-n$

$= 11\times 36-3$

$= 396-3$

$= 393$

$t - \frac{1}{t} = |13-16| = 3$

71. If $x^2 + 13x + 39 = 0$, then $(x + 8)^5 - \frac{1}{(x+8)^5}$

t

= ?

(a) 393 (b) 396

(c) 392 (d) 394

$t^5 - \frac{1}{t^5} = (3^2+2)(3^3+3\times3)-3$

$= 393$

72. If $x^2 - 12x + 33 = 0$, then what is the value of $(x-4)^4 + \frac{1}{(x-4)^4} = ?$

यदि $x^2 - 12x + 33 = 0$ तो $(x-4)^4 + \frac{1}{(x-4)^4} =$ का मान क्या है?

(a) 227 (b) 326

(c) 167 (d) 194

$x - 4 = t$

$\therefore x = t + 4$

$(t+4)^2 - 12(t+4) + 33 = 0$

$\Rightarrow t^2 + 16 + 8t - 12t - 48 + 33 = 0$

$\Rightarrow t^2 - 4t + 1 = 0$

$\Rightarrow t - 4 + \frac{1}{t} = 0$

$\therefore t + \frac{1}{t} = 4$

$t^4 + \frac{1}{t^4} = (n^2 - 2)^2 - 2$

$= 196 - 2$

$= 194$

$t + \frac{1}{t} = |12 - 8| = 4$

$t^4 + \frac{1}{t^4} = (n^2 - 2)^2 - 2$

$= (14)^2 - 2$

$= 194$

72. If $x^2 - 12x + 33 = 0$, then what is the value of $(x - 4)^4 + \frac{1}{(x - 4)^4} = ?$

यदि $x^2 - 12x + 33 = 0$ तो $(x - 4)^4 + \frac{1}{(x - 4)^4} =$ का मान क्या है?

(a) 227 (b) 326

(c) 167 (d) 194

$t + \frac{1}{t} = |12 - 8| = 4$

$t^4 + \frac{1}{t^4} = (n^2 - 2)^2 - 2$

$= (14)^2 - 2$

$= 194$

72. If $x^2 - 12x + 33 = 0$, then what is the value of $(x - 4)^4 + \frac{1}{(x - 4)^4} = ?$

यदि $x^2 - 12x + 33 = 0$ तो $(x - 4)^4 + \frac{1}{(x - 4)^4} =$ का मान क्या है?

(a) 227 (b) 326

(c) 167 (d) 194

73. If $(x - \cancel{a})(x - b) = 1$ and $\cancel{a} - b + 5 = 0$, then

$$(x - \cancel{a})^3 + \frac{1}{(x - \cancel{a})^3} = ?$$

(V.V.Imp)

यदि $(x \quad)(x - 5) = 1$ और तो

$$(x \quad)^3 - \frac{1}{(x \quad)^3} = ?$$

(a) 110 (b) 1

(c) 125 (d) 140

$x(x-b)=1$

$\Rightarrow x-b=\frac{1}{x}$

$\Rightarrow x-\frac{1}{x}=5$

$-b+5=0$

$5=b$

$\therefore x^3-\frac{1}{x^3}=(n^3+3n)$

$=125+15$

$=140$

$$* \; x - \frac{1}{x} = 3 \; (n)$$

$$* \; x^5 - \frac{1}{x^5} = (n^2+2)(n^3+3n) - n$$

$$= 11 \times 36 - 3$$

$$= 396 - 3$$

$$= \underline{\underline{393}}$$

$$* \; x^{100} - \frac{1}{x^{100}} = 3$$

$$* \; x^{500} - \frac{1}{x^{500}} = (n^2+2)(n^3+3n) - n$$

$$= 11 \times 36 - 3$$

$$= \underline{393}$$

74. If $x^{\cancel{5}\,1} - \frac{1}{x^{\cancel{5}\,1}} = \sqrt{7}$, then find $x^{\cancel{35}\,7} - \frac{1}{x^{\cancel{35}\,7}} = ?$

(a) $791\sqrt{7}$ (b) $789\sqrt{7}$

(c) $790\sqrt{7}$ (d) $792\sqrt{7}$

$x - \frac{1}{x} = \sqrt{7}$

$\therefore x^2 + \frac{1}{x^2} = (\sqrt{7})^2 + 2 = 9$

$x^4 + \frac{1}{x^4} = (9)^2 - 2 = 79$

$\therefore x^3 - \frac{1}{x^3} = n^3 + 3n$

$= 7\sqrt{7} + 3\sqrt{7}$

$= 10\sqrt{7}$

$x^7 - \frac{1}{x^7} = \left(x^3 - \frac{1}{x^3}\right)\left(x^4 + \frac{1}{x^4}\right) + \left(x - \frac{1}{x}\right)$

$= 10\sqrt{7} \times 79 + \sqrt{7}$

$= 790\sqrt{7} + \sqrt{7}$

$= 791\sqrt{7}$

75. If $3\sqrt{x} + \frac{1}{2\sqrt{x}} = 3, x > 0$ then find $x^2(18x^2 - 7)$?

यदि $3\sqrt{x} + \frac{1}{2\sqrt{x}} = 3, x > 0$ है ,तो $x^2(18x^2 - 7)$ ज्ञात कीजिए?

(a) $-\frac{1}{36}$ (b) $-\frac{2}{63}$

(c) $-\frac{1}{72}$ (d) $-\frac{8}{81}$

$$\left(3\sqrt{x} + \frac{1}{2\sqrt{x}}\right)^2 = (3)^2$$

$$9x + \frac{1}{4x} + 2 \times 3\sqrt{x} \times \frac{1}{2\sqrt{x}} = 9$$

$$\left(9x + \frac{1}{4x}\right)^2 = (6)^2$$

$$81x^2 + \frac{1}{16x^2} + 2 \times 9x \times \frac{1}{4x} = 36$$

$$\frac{2}{9} \times 81x^2 + \frac{2}{9} \times \frac{1}{16x^2} + \frac{9}{2} \times \frac{2}{9} = 36 \times \frac{2}{9}$$

$$\Rightarrow 18x^2 + \frac{1}{72x^2} + 1 = 8$$

$$\Rightarrow (18x^2 - 7) = -\frac{1}{72x^2}$$

$$x^2(18x^2 - 7) = -\frac{1}{72}$$

$9x + \frac{1}{4x} = 3^2 - 2 \times 3 \times \frac{1}{2} = 6$

75. If $3\sqrt{x} + \frac{1}{2\sqrt{x}} = 3$, $x > 0$ then find $x^2(18x^2 - 7)$?

यदि $3\sqrt{x} + \frac{1}{2\sqrt{x}} = 3$, $x > 0$ है ,तो $x^2(18x^2 - 7)$ ज्ञात कीजिए?

$\frac{2}{9}\left(81x^2 + \frac{1}{16x^2}\right) = \left(6^2 - 2 \times 9 \times \frac{1}{4 \times 2}\right)\frac{2}{9}$

$\Rightarrow 18x^2 + \frac{1}{72x^2} = 8 - 1$

$\Rightarrow 18x^2 - 7 = -\frac{1}{72x^2}$

$\Rightarrow x^2(18x^2 - 7) = -\frac{1}{72}$

(a) $-\frac{1}{36}$

(b) $-\frac{2}{63}$

(c) $-\frac{1}{72}$ ✓

(d) $-\frac{8}{81}$